Trigonometry/Precalculus Name ______________________

Quarter 4 Exam Study Guide SOLUTIONS Date ____________ Pd ______

1a. $a = 4,\ \ b = 3,\ \ c^2 = a^2 - b^2 \Rightarrow c^2 = 16 - 9 \Rightarrow c^2 = 7 \Rightarrow c = \sqrt{7}$

equation: $\dfrac{(x+3)^2}{16} + \dfrac{(y-2)^2}{9} = 1$

center: $(-3,\ 2)$ vertices: $(-7,\ 2)$ and $(1,\ 2)$

foci: $(-3-\sqrt{7},\ 2)$ and $(-3+\sqrt{7},\ 2)$ eccentricity: $\sqrt{7}/4$

1b. $a = 6,\ \ b = 4,\ \ c^2 = a^2 - b^2 \Rightarrow c^2 = 36 - 16 \Rightarrow c^2 = 20 \Rightarrow c = 2\sqrt{5}$

equation: $\dfrac{(y-1)^2}{36} + \dfrac{(x+4)^2}{16} = 1$

center: $(-4,\ 1)$ vertices: $(-4,\ -5)$ and $(-4,\ 7)$

foci: $(-4,\ 1-2\sqrt{5})$ and $(-4,\ 1+2\sqrt{5})$ eccentricity: $\sqrt{5}/3$

1c. $a = 5,\ \ c = 4,\ \ c^2 = a^2 - b^2 \Rightarrow 16 = 25 - b^2 \Rightarrow b^2 = 9 \Rightarrow b = 3$

equation: $\dfrac{(y+1)^2}{25} + \dfrac{(x-3)^2}{9} = 1$

center: $(3,\ -1)$ vertices: $(3,\ -6)$ and $(3,\ 4)$

foci: $(3,\ -5)$ and $(3,\ 3)$ eccentricity: $4/5$

1d. $a = 3,\ \ c = 1,\ \ c^2 = a^2 - b^2 \Rightarrow 1 = 9 - b^2 \Rightarrow b^2 = 8 \Rightarrow b = 2\sqrt{2}$

equation: $\dfrac{(x-2)^2}{9} + \dfrac{(y+1)^2}{8} = 1$

center: $(2,\ -1)$ vertices: $(-1,\ -1)$ and $(5,\ -1)$

foci: $(1,\ -1)$ and $(3,\ -1)$ eccentricity: $1/3$

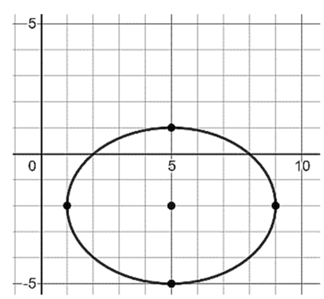
2a. $9x^2 - 90x + 16y^2 + 64y = -145$

$9(x^2 - 10x + \underline{25}) + 16(y^2 + 4y + \underline{4}) = -145 + 9(25) + 16(4)$

$9(x-5)^2 + 16(y+2)^2 = 144$

$\dfrac{9(x-5)^2}{144} + \dfrac{16(y+2)^2}{144} = 1$

$a = 4,\ \ b = 3,\ \ c^2 = a^2 - b^2 \Rightarrow c^2 = 16 - 9 \Rightarrow c^2 = 7 \Rightarrow c = \sqrt{7}$



equation: $\dfrac{(x-5)^2}{16} + \dfrac{(y+2)^2}{9} = 1$

center: $(5,\ -2)$

vertices: $(1,\ -2)$ and $(9,\ -2)$

foci: $(5-\sqrt{7},\ -2)$ and $(5+\sqrt{7},\ -2)$

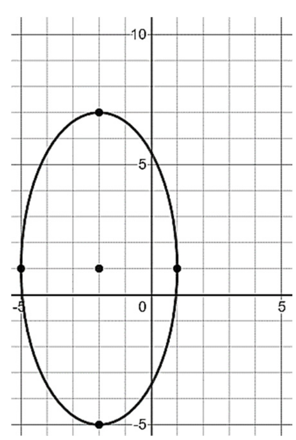
2b. $4x^2 + 16x + y^2 - 2y = 19$

$4(x^2 + 4x + \underline{4}) + (y^2 - 2y + \underline{1}) = 19 + 4(4) + 1$

$4(x+2)^2 + (y-1)^2 = 36$

$\dfrac{(y-1)^2}{36} + \dfrac{4(x+2)^2}{36} = 1$

$a = 6,\ \ b = 3,\ \ c^2 = a^2 - b^2 \Rightarrow c^2 = 36 - 9 \Rightarrow c^2 = 27 \Rightarrow c = 3\sqrt{3}$



equation: $\dfrac{(y-1)^2}{36} + \dfrac{(x+2)^2}{9} = 1$

center: $(-2,\ 1)$

vertices: $(-2,\ -5)$ and $(-2,\ 7)$

foci: $(-2,\ 1-3\sqrt{3})$ and $(-2,\ 1+3\sqrt{3})$

3a. $a=2,\ c=4,\ c^2=a^2+b^2 \Rightarrow 16=4+b^2 \Rightarrow b^2=12 \Rightarrow b=2\sqrt{3}$

equation: $\dfrac{(x+4)^2}{4}-\dfrac{(y+2)^2}{12}=1$

center: $(-4,\ -2)$

vertices: $(-6,\ -2)$ and $(-2,\ -2)$

foci: $(-8,\ -2)$ and $(0,\ -2)$

asymptotes: $y=-2\pm\sqrt{3}(x+4)$

3b. $c=5,\ \dfrac{a}{b}=\dfrac{4}{3} \Rightarrow a=\dfrac{4}{3}b,\ c^2=a^2+b^2 \Rightarrow 25=\dfrac{16}{9}b^2+b^2 \Rightarrow 25=\dfrac{25}{9}b^2 \Rightarrow b^2=9 \Rightarrow b=3,\ a=\dfrac{4}{3}(3)=4$

equation: $\dfrac{(y-4)^2}{16}-\dfrac{(x-1)^2}{9}=1$

center: $(1,\ 4)$

vertices: $(1,\ 0)$ and $(1,\ 8)$

foci: $(1,\ -1)$ and $(1,\ 9)$

asymptotes: $y=4\pm\dfrac{4}{3}(x-1)$

3c. $a=2,\ \dfrac{a}{b}=\dfrac{1}{2} \Rightarrow \dfrac{2}{b}=\dfrac{1}{2} \Rightarrow b=4,\ c^2=a^2+b^2 \Rightarrow c^2=4+16 \Rightarrow c^2=20 \Rightarrow c=2\sqrt{5}$

equation: $\dfrac{(y-1)^2}{4}-\dfrac{(x-3)^2}{16}=1$

center: $(3,\ 1)$

vertices: $(3,\ -1)$ and $(3,\ 3)$

foci: $(3,\ 1-2\sqrt{5})$ and $(3,\ 1+2\sqrt{5})$

asymptotes: $y=1\pm\dfrac{1}{2}(x-3)$

3d. $a=2,\ \dfrac{(x+4)^2}{4}-\dfrac{(y-1)^2}{b^2}=1 \Rightarrow \dfrac{(2+4)^2}{4}-\dfrac{(5-1)^2}{b^2}=1 \Rightarrow 9-\dfrac{16}{b^2}=1 \Rightarrow \dfrac{16}{b^2}=8 \Rightarrow b^2=2 \Rightarrow b=\sqrt{2}$

$c^2=a^2+b^2 \Rightarrow c^2=4+2 \Rightarrow c^2=6 \Rightarrow c=\sqrt{6}$

equation: $\dfrac{(x+4)^2}{4}-\dfrac{(y-1)^2}{2}=1$

center: $(-4,\ 1)$

vertices: $(-6,\ 1)$ and $(-2,\ 1)$

foci: $(-4-\sqrt{6},\ 1)$ and $(-4+\sqrt{6},\ 1)$

asymptotes: $y=1\pm\dfrac{\sqrt{2}}{2}(x+4)$

4a.

$$4x^2-40x-9y^2-54y=17$$
$$4(x^2-10x+\underline{25})-9(y^2+6y+\underline{9})=17+4(25)-9(9)$$
$$4(x-5)^2-9(y+3)^2=36$$
$$\frac{4(x-5)^2}{36}-\frac{9(y+3)^2}{36}=1$$

$a=3,\ b=2,\ c^2=a^2+b^2 \Rightarrow c^2=9+4 \Rightarrow c^2=13 \Rightarrow c=\sqrt{13}$

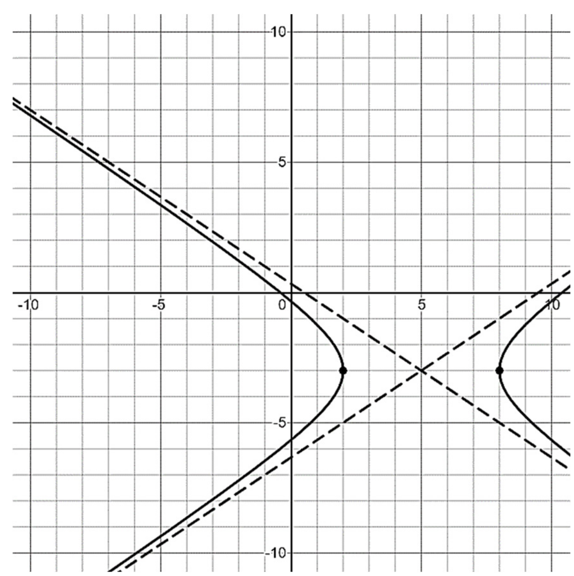
equation: $\dfrac{(x-5)^2}{9}-\dfrac{(y+3)^2}{4}=1$

center: $(5,\ -3)$

vertices: $(2,\ -3)$ and $(8,\ -3)$

foci: $(5-\sqrt{13},\ -3)$ and $(5+\sqrt{13},\ -3)$

asymptotes: $y=-3\pm\dfrac{2}{3}(x-5)$



4b. $$y^2 - 2y - 4x^2 - 16x = 31$$

$$(y^2 - 2y + \underline{1}) - 4(x^2 + 4x + \underline{4}) = 31 + 1 - 4(4)$$

$$(y-1)^2 - 4(x+2)^2 = 16$$

$$\frac{(y-1)^2}{16} - \frac{4(x+2)^2}{16} = 1$$

$a = 4,\ \ b = 2,\ \ c^2 = a^2 + b^2 \Rightarrow c^2 = 16 + 4 \Rightarrow c^2 = 20 \Rightarrow c = 2\sqrt{5}$

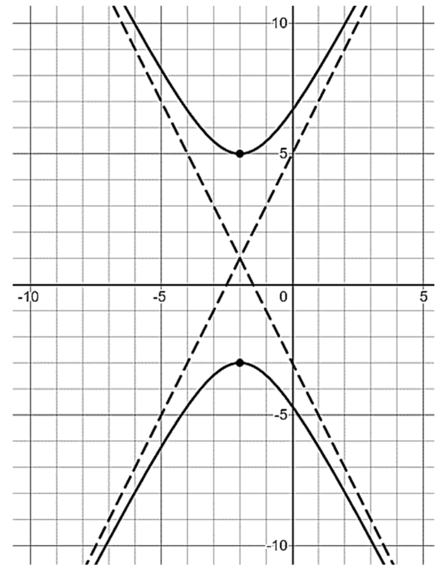
equation: $\dfrac{(y-1)^2}{16} - \dfrac{(x+2)^2}{4} = 1$

center: $(-2,\ 1)$

vertices: $(-2,\ -3)$ and $(-2,\ 5)$

foci: $(-2,\ 1-2\sqrt{5})$ and $(-2,\ 1+2\sqrt{5})$

asymptotes: $y = 1 \pm 2(x+2)$



5a. Note: Because the graph is linear, only two points are necessary to draw the graph.

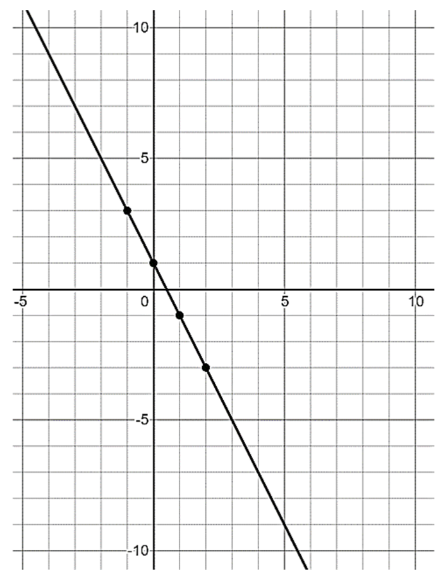
t	0	2	4	6
x	–1	0	1	2
y	3	1	–1	–3

$x = 0.5t - 1 \Rightarrow 0.5t = x + 1 \Rightarrow t = 2x + 2$

$y = 3 - t \Rightarrow y = 3 - (2x + 2) \Rightarrow y = 3 - 2x - 2$

equation: $y = -2x + 1$

domain: all real numbers



5b. Domain for t: $t + 1 \geq 0 \Rightarrow t \geq -1$

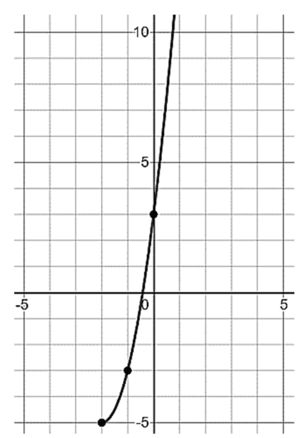
t	–1	0	3
x	–2	–1	0
y	–5	–3	3

$x = \sqrt{t+1} - 2 \Rightarrow \sqrt{t+1} = x + 2 \Rightarrow t + 1 = (x+2)^2 \Rightarrow t = x^2 + 4x + 4 - 1 \Rightarrow t = x^2 + 4x + 3$

$y = 2t - 3 \Rightarrow y = 2(x^2 + 4x + 3) - 3 \Rightarrow y = 2x^2 + 8x + 6 - 3$

equation: $y = 2x^2 + 8x + 3$

domain: $x \geq -2$



5c. Domain for t: $t-1 \neq 0 \Rightarrow t \neq 1$

t	–5	–4	–3	–2	–1	0	1/2	2/3	3/4	4/5	5/6
x	5/6	4/5	3/4	2/3	1/2	0	–1	–2	–3	–4	–5
y	–10	–8	–6	–4	–2	0	1	4/3	3/2	8/5	5/3

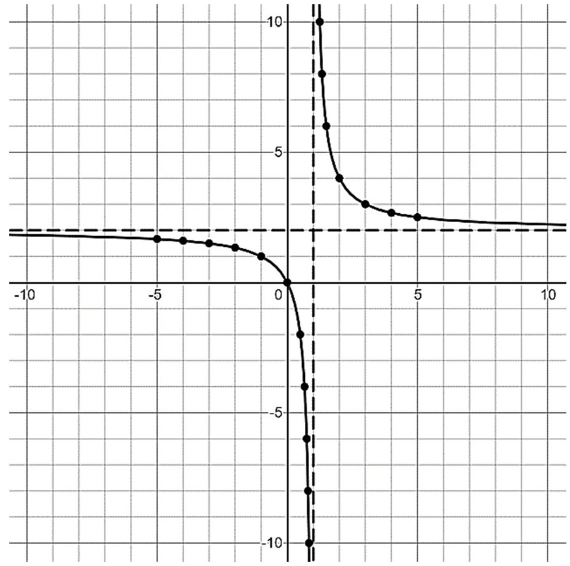
t	5/4	4/3	3/2	2	3	4	5
x	5	4	3	2	3/2	4/3	5/4
y	5/2	8/3	3	4	6	8	10

$$x=\frac{t}{t-1} \Rightarrow x(t-1)=t \Rightarrow xt-x=t \Rightarrow xt-t=x \Rightarrow t(x-1)=x \Rightarrow t=\frac{x}{x-1}$$

$$y=2t \Rightarrow y=2\left(\frac{x}{x-1}\right)$$

equation: $y=\dfrac{2x}{x-1}$

domain: $x \neq 1$



5d.

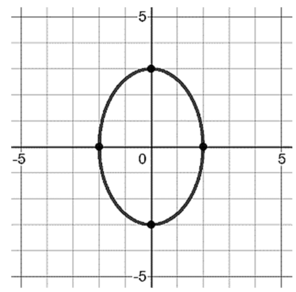
t	0	$\pi/2$	π	$3\pi/2$	2π
x	2	0	–2	0	2
y	0	3	0	–3	0

$$x=2\cos(\theta) \Rightarrow \cos(\theta)=\frac{x}{2},\ \ y=3\sin(\theta) \Rightarrow \sin(\theta)=\frac{y}{3}$$

$$\sin^2(\theta)+\cos^2(\theta)=1 \Rightarrow \left(\frac{y}{3}\right)^2+\left(\frac{x}{2}\right)^2=1$$

equation: $\dfrac{y^2}{9}+\dfrac{x^2}{4}=1$

domain: $-2 \le x \le 2$



6a. $\dfrac{6!}{(3!)(8!)}=\dfrac{6!}{(3!)(8\cdot 7\cdot 6!)}=\dfrac{1}{(3!)(8\cdot 7)}=\dfrac{1}{(6)(8\cdot 7)}=\dfrac{1}{336}$

6b. $\dfrac{(7!)(3!)}{(4!)(5!)}=\dfrac{(7\cdot 6\cdot 5!)(3!)}{(4\cdot 3!)(5!)}=\dfrac{7\cdot 6}{4}=\dfrac{7\cdot 3}{2}=\dfrac{21}{2}$

6c. $\dfrac{(n+2)!}{n!}=\dfrac{(n+2)(n+1)(n!)}{n!}=(n+2)(n+1)=n^2+3n+2$

6d. $\dfrac{(n-3)!}{n!}=\dfrac{(n-3)!}{n(n-1)(n-2)(n-3)!}=\dfrac{1}{n(n-1)(n-2)}=\dfrac{1}{n^3-3n^2+2n}$

7a. $\dfrac{1}{1}+\dfrac{4}{3}+\dfrac{9}{9}+\dfrac{16}{27}+\cdots=\dfrac{(0+1)^2}{3^0}+\dfrac{(1+1)^2}{3^1}+\dfrac{(2+1)^2}{3^2}+\dfrac{(3+1)^2}{3^3}+\cdots=\sum_{n=0}^{\infty}\dfrac{(n+1)^2}{3^n}$

7b. $\dfrac{1}{3}-\dfrac{1}{5}+\dfrac{2}{7}-\dfrac{6}{9}+\dfrac{24}{11}-\dfrac{120}{13}+\cdots=\dfrac{0!}{2(0)+3}-\dfrac{1!}{2(1)+3}+\dfrac{2!}{2(2)+3}-\dfrac{3!}{2(3)+3}+\dfrac{4!}{2(4)+3}-\dfrac{5!}{2(5)+3}+\cdots=\sum_{n=0}^{\infty}\dfrac{(-1)^n n!}{2n+3}$

7c. $\dfrac{3}{20}+\dfrac{6}{17}+\dfrac{11}{14}+\dfrac{18}{11}+\dfrac{27}{8}+\cdots=\dfrac{1^2+2}{23-3(1)}+\dfrac{2^2+2}{23-3(2)}+\dfrac{3^2+2}{23-3(3)}+\dfrac{4^2+2}{23-3(4)}+\dfrac{5^2+2}{23-3(5)}+\cdots=\sum_{n=1}^{\infty}\dfrac{n^2+2}{23-3n}$

7d. $\dfrac{2}{1}-\dfrac{4}{2}+\dfrac{8}{24}-\dfrac{16}{720}+\cdots=\dfrac{2^1}{(2\cdot 1-2)!}-\dfrac{2^2}{(2\cdot 2-2)!}+\dfrac{2^3}{(2\cdot 3-2)!}-\dfrac{2^4}{(2\cdot 4-2)!}+\cdots=\sum_{n=1}^{\infty}\dfrac{(-1)^{n+1}2^n}{(2n-2)!}$

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8a. $-17-13-9-5-1+3+7+11+15=\frac{9}{2}(-17+15)=\frac{9}{2}(-2)=-9$

8b. $3+5+7+9+\cdots+31=\frac{15}{2}(3+31)=\frac{15}{2}(34)=15(17)=255$

8c. $\sum_{n=1}^{6}(4n-11)=\frac{6}{2}(-7+13)=3(6)=18$

8d. $\sum_{n=0}^{8}(23-2n)=\frac{9}{2}(23+7)=\frac{9}{2}(30)=9(15)=135$

9a. $1-2+4-8+16=\frac{1\left(1-(-2)^5\right)}{1-(-2)}=\frac{1\left(1-(-32)\right)}{1-(-2)}=\frac{1(1+32)}{1+2}=\frac{33}{3}=11$

9b. $81+27+9+3+1=\frac{81\left(1-(1/3)^5\right)}{1-(1/3)}=\frac{81\left(1-(1/243)\right)}{1-(1/3)}=\frac{81(242/243)}{2/3}=\frac{242/3}{2/3}=\frac{242}{2}=121$

9c. $\sum_{n=0}^{6}5\left(-\frac{1}{2}\right)^n=\frac{5\left(1-(-1/2)^7\right)}{1-(-1/2)}=\frac{5\left(1-(-1/128)\right)}{1-(-1/2)}=\frac{5(1+1/128)}{1+1/2}=\frac{5(129/128)}{3/2}\cdot\frac{128}{128}=\frac{5(129)}{3(64)}=\frac{5(43)}{64}=\frac{215}{64}$

9d. $\sum_{n=2}^{7}2(3)^n=\frac{18(1-3^6)}{1-3}=\frac{18(1-729)}{1-3}=\frac{18(-728)}{-2}=-9(728)=6552$

10a. $10-\frac{10}{3}+\frac{10}{9}-\frac{10}{27}+\cdots=\sum_{n=0}^{\infty}10\left(-\frac{1}{3}\right)^n=\frac{10}{1-(-1/3)}=\frac{10}{1+1/3}=\frac{10}{4/3}\cdot\frac{3}{3}=\frac{30}{4}=\frac{15}{2}$

10b. $1+\frac{3}{2}+\frac{9}{4}+\frac{27}{8}+\cdots=\sum_{n=0}^{\infty}\left(\frac{3}{2}\right)^n$ does not have a sum, since $\left|\frac{3}{2}\right|=\frac{3}{2}\geq 1$.

10c. $\sum_{n=0}^{\infty}12\left(-\frac{6}{5}\right)^n$ does not have a sum, since $\left|-\frac{6}{5}\right|=\frac{6}{5}\geq 1$.

10d. $\sum_{n=1}^{\infty}12\left(\frac{2}{3}\right)^n=\frac{8}{1-(2/3)}=\frac{8}{1/3}\cdot\frac{3}{3}=\frac{24}{1}=24$

11a.

x	–3.1	–3.01	–3.001	–3	–2.999	–2.99	–2.9
$f(x)$	–5.94736	–5.5427	–5.50425	undef	–5.49575	–5.4577	–5.0952

$\lim_{x\to -3} f(x)=-5.5$. As x approaches -3 from the left and the right, $f(x)$ approaches -5.5.

11b.

x	–3.1	–3.01	–3.001	–3	–2.999	–2.99	–2.9
$f(x)$	1.94868	1.99498	1.99949	undef	2.00049	2.00498	2.0488

$\lim_{x\to -3} f(x)=2$. As x approaches -3 from the left and the right, $f(x)$ approaches 2.

11c.

x	–3.1	–3.01	–3.001	–3	–2.999	–2.99	–2.9
$f(x)$	0.55166	0.5050	0.5005	undef	0.4995	0.4950	0.45166

$\lim_{x\to -3} f(x)=0.5$. As x approaches -3 from the left and the right, $f(x)$ approaches 0.5.

12a. $\lim_{x\to -3} f(x)$ DNE. As x approaches -3 from the left, $f(x)$ approaches -2. As x approaches -3 from the right, $f(x)$ approaches 0.

12b. $\lim_{x\to 1} f(x)=4$. As x approaches 1 from the left and the right, $f(x)$ approaches 4.

12c. $\lim_{x\to 2} f(x)$ DNE. As x approaches 2 from the left, $f(x)$ decreases without bound. (OR)
As x approaches 2 from the right, $f(x)$ increases without bound.

12d. $\lim_{x\to 4} f(x)=2$. As x approaches 4 from the left and the right, $f(x)$ approaches 2.

13a. $\lim_{x\to 3}(2x^2-7x+3)=2(3)^2-7(3)+3=0$ and $\lim_{x\to 3}(x^3-27)=3^3-27=0$

$$\lim_{x\to 3}\frac{2x^2-7x+3}{x^3-27}=\lim_{x\to 3}\frac{(2x-1)(x-3)}{(x-3)(x^2+3x+9)}=\lim_{x\to 3}\frac{2x-1}{x^2+3x+9}=\frac{2(3)-1}{3^2+3(3)+9}=\frac{5}{27}$$

13b. $\lim_{x\to -4}(x^2+6x+8)=(-4)^2+6(-4)+8=0$ and $\lim_{x\to -4}(x^2+x-12)=(-4)^2+(-4)-12=0$

$$\lim_{x\to -4}\frac{x^2+6x+8}{x^2+x-12}=\lim_{x\to -4}\frac{(x+4)(x+2)}{(x+4)(x-3)}=\lim_{x\to -4}\frac{x+2}{x-3}=\frac{-4+2}{-4-3}=\frac{-2}{-7}=\frac{2}{7}$$

13c. $\lim_{x\to 2}(x-2)=2-2=0$ and $\lim_{x\to 2}(\sqrt{x+7}-3)=\sqrt{2+7}-3=0$

$$\lim_{x\to 2}\frac{x-2}{\sqrt{x+7}-3}=\lim_{x\to 2}\frac{x-2}{\sqrt{x+7}-3}\cdot\frac{\sqrt{x+7}+3}{\sqrt{x+7}+3}=\lim_{x\to 2}\frac{(x-2)(\sqrt{x+7}+3)}{x+7-9}=\lim_{x\to 2}\frac{(x-2)(\sqrt{x+7}+3)}{x-2}=\lim_{x\to 2}(\sqrt{x+7}+3)=\sqrt{2+7}+3=6$$

13d. $\lim_{x\to -3}(\sqrt{x+4}-1)=\sqrt{-3+4}-1=0$ and $\lim_{x\to -3}(x+3)=-3+3=0$

$$\lim_{x\to -3}\frac{\sqrt{x+4}-1}{x+3}=\lim_{x\to -3}\frac{\sqrt{x+4}-1}{x+3}\cdot\frac{\sqrt{x+4}+1}{\sqrt{x+4}+1}=\lim_{x\to -3}\frac{x+4-1}{(x+3)(\sqrt{x+4}+1)}=\lim_{x\to -3}\frac{x+3}{(x+3)(\sqrt{x+4}+1)}=\lim_{x\to -3}\frac{1}{\sqrt{x+4}+1}=\frac{1}{\sqrt{-3+4}+1}=\frac{1}{2}$$

13e. $\lim_{x\to 1}\left(\frac{1}{x+4}-\frac{1}{5}\right)=\frac{1}{1+4}-\frac{1}{5}=0$ and $\lim_{x\to 1}(x-1)=1-1=0$

$$\lim_{x\to 1}\frac{\frac{1}{x+4}-\frac{1}{5}}{x-1}=\lim_{x\to 1}\frac{\frac{1}{x+4}-\frac{1}{5}}{x-1}\cdot\frac{5(x+4)}{5(x+4)}=\lim_{x\to 1}\frac{1(5)-1(x+4)}{5(x-1)(x+4)}=\lim_{x\to 1}\frac{5-x-4}{5(x-1)(x+4)}=\lim_{x\to 1}\frac{1-x}{5(x-1)(x+4)}=\lim_{x\to 1}\frac{-1(x-1)}{5(x-1)(x+4)}$$

$$=\lim_{x\to 1}\frac{-1}{5(x+4)}=-\frac{1}{5(1+4)}=-\frac{1}{25}$$

13f. $\lim_{x\to -3}\left(\frac{1}{4-x}-\frac{1}{7}\right)=\frac{1}{4-(-3)}-\frac{1}{7}=0$ and $\lim_{x\to -3}(x+3)=-3+3=0$

$$\lim_{x\to -3}\frac{\frac{1}{4-x}-\frac{1}{7}}{x+3}=\lim_{x\to -3}\frac{\frac{1}{4-x}-\frac{1}{7}}{x+3}\cdot\frac{7(4-x)}{7(4-x)}=\lim_{x\to -3}\frac{1(7)-1(4-x)}{7(x+3)(4-x)}=\lim_{x\to -3}\frac{7-4+x}{7(x+3)(4-x)}=\lim_{x\to -3}\frac{x+3}{7(x+3)(4-x)}$$

$$=\lim_{x\to -3}\frac{1}{7(4-x)}=\frac{1}{7\left(4-(-3)\right)}=\frac{1}{49}$$