Trigonometry/Precalculus Name ______________________

Quarter 2 Exam Study Guide SOLUTIONS Date ______________ Pd ____

1a. $B = 83.2°$, $C = 90°$, $b = 181.9816$ or 181.9817, $c = 183.2708$ or 183.2709

$B = 180° - A - C = 180° - 6.8° - 90° = 83.2°$

$\cot(A) = \dfrac{b}{a} \Rightarrow b = a\cot(A) = 21.7\cot(6.8°) = 181.98166695$

$\csc(A) = \dfrac{c}{a} \Rightarrow c = a\csc(A) = 21.7\csc(6.8°) = 183.27088$

1b. $B = 62°$, $C = 90°$, $a = 4.7853$ or 4.7854, $c = 10.1931$

$B = 180° - A - C = 180° - 28° - 90° = 62°$

$\tan(A) = \dfrac{a}{b} \Rightarrow a = b\tan(A) = 9\tan(28°) = 4.78538$

$\sec(A) = \dfrac{c}{b} \Rightarrow c = b\sec(A) = 9\sec(28°) = 10.1931$

1c. $A = 29.3271°$ or $29.3272°$, $B = 60.6728°$, $C = 90°$, $b = 48.5077$

$a^2 + b^2 = c^2 \Rightarrow b^2 = c^2 - a^2 \Rightarrow b = \sqrt{c^2 - a^2} = \sqrt{49^2 - 24^2} = 48.5077$

$\sin(A) = \dfrac{a}{c} \Rightarrow A = \sin^{-1}\left(\dfrac{a}{c}\right) = \sin^{-1}\left(\dfrac{24}{49}\right) = 29.327168765$

$B = 180° - A - C = 180° - 29.327168765° - 90° = 60.6728°$

2. $\cot(23°) = \dfrac{x}{200} \Rightarrow x = 200\cot(23°) = 471.1704$ or 471.1705 feet

3. $\sin(50°) = \dfrac{y}{450} \Rightarrow y = 450\sin(50°) = 344.7199$ or 344.7200 feet

4. $\cot(32°) = \dfrac{x}{h} \Rightarrow x = h\cot(32°)$ and $\cot(35°) = \dfrac{x - 1000}{h} \Rightarrow x - 1000 = h\cot(35°) \Rightarrow x = 1000 + h\cot(35°)$

$h\cot(32°) = 1000 + h\cot(35°) \Rightarrow h\cot(32°) - h\cot(35°) = 1000 \Rightarrow h = \dfrac{1000}{\cot(32°) - \cot(35°)} = 5807.6554$ or 5807.6555 feet

5a. $\dfrac{\sin(\theta)}{\tan(\theta)} = \sin(\theta)\cot(\theta) = \dfrac{\sin(\theta)}{1} \cdot \dfrac{\cos(\theta)}{\sin(\theta)} = \cos(\theta)$

5b. $\tan(\theta) + \cot(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)} + \dfrac{\cos(\theta)}{\sin(\theta)} = \dfrac{\sin^2(\theta) + \cos^2(\theta)}{\sin(\theta)\cos(\theta)} = \dfrac{1}{\sin(\theta)\cos(\theta)} = \dfrac{1}{\cos(\theta)} \cdot \dfrac{1}{\sin(\theta)} = \sec(\theta)\csc(\theta)$

5c. $(\sin(x) + \cos(x))^2 = \sin^2(x) + 2\sin(x)\cos(x) + \cos^2(x) = 1 + 2\sin(x)\cos(x)$

5d. $(1 - \cos(\beta))(1 + \cos(\beta)) = 1 - \cos^2(\beta) = \sin^2(\beta) = \dfrac{1}{\csc^2(\beta)}$

5e. $(\tan(x) + \cot(x))^2 = \tan^2(x) + 2\tan(x)\cot(x) + \cot^2(x) = \sec^2(x) - 1 + 2 + \csc^2(x) - 1 = \sec^2(x) + \csc^2(x)$

5f. $\dfrac{\sec(x) + \csc(x)}{\tan(x) + \cot(x)} = \dfrac{\frac{1}{\cos(x)} + \frac{1}{\sin(x)}}{\frac{\sin(x)}{\cos(x)} + \frac{\cos(x)}{\sin(x)}} \cdot \dfrac{\sin(x)\cos(x)}{\sin(x)\cos(x)} = \dfrac{\sin(x) + \cos(x)}{\sin^2(x) + \cos^2(x)} = \dfrac{\sin(x) + \cos(x)}{1} = \sin(x) + \cos(x)$

5g. $\dfrac{\sin^3(x) + \cos^3(x)}{\sin(x) + \cos(x)} = \dfrac{(\sin(x) + \cos(x))(\sin^2(x) - \sin(x)\cos(x) + \cos^2(x))}{\sin(x) + \cos(x)} = \sin^2(x) - \sin(x)\cos(x) + \cos^2(x) = 1 - \sin(x)\cos(x)$

5h. $\dfrac{\cos(\theta)}{1 - \sin(\theta)} = \dfrac{\cos(\theta)}{1 - \sin(\theta)} \cdot \dfrac{1 + \sin(\theta)}{1 + \sin(\theta)} = \dfrac{\cos(\theta)(1 + \sin(\theta))}{1 - \sin^2(\theta)} = \dfrac{\cos(\theta)(1 + \sin(\theta))}{\cos^2(\theta)} = \dfrac{1 + \sin(\theta)}{\cos(\theta)} = \dfrac{1}{\cos(\theta)} + \dfrac{\sin(\theta)}{\cos(\theta)} = \sec(\theta) + \tan(\theta)$

6a. $\sqrt{2}\sin(\theta)+1=0 \Rightarrow \sin(\theta)=-\frac{1}{\sqrt{2}}=-\frac{\sqrt{2}}{2} \Rightarrow \theta=\frac{5\pi}{4}+2\pi k,\ \frac{7\pi}{4}+2\pi k,\ k\in\mathbb{Z}$

6b. $3\tan^2(\theta)-1=0 \Rightarrow \tan^2(\theta)=\frac{1}{3} \Rightarrow \tan(\theta)=\pm\frac{1}{\sqrt{3}} \Rightarrow \theta=\frac{\pi}{6}+2k,\ \frac{5\pi}{6}+2\pi k,\ \frac{7\pi}{6}+2\pi k,\ \frac{11\pi}{6}+2\pi k,\ k\in\mathbb{Z}$

Also acceptable: $\theta=\frac{\pi}{6}+\pi k,\ \frac{5\pi}{6}+\pi k,\ k\in\mathbb{Z}$

6c. $\sec^2(\theta)-2=0 \Rightarrow \sec^2(\theta)=2 \Rightarrow \sec(\theta)=\pm\sqrt{2} \Rightarrow \cos(\theta)=\pm\frac{1}{\sqrt{2}}=\pm\frac{\sqrt{2}}{2} \Rightarrow \theta=\frac{\pi}{4}+2\pi k,\ \frac{3\pi}{4}+2\pi k,\ \frac{5\pi}{4}+2\pi k,\ \frac{7\pi}{4}+2\pi k,\ k\in\mathbb{Z}$

Also acceptable: $\theta=\frac{\pi}{4}+\frac{\pi}{2}k,\ k\in\mathbb{Z}$

6d. $2s^2-s-1=0 \Rightarrow (2s+1)(s-1)=0 \Rightarrow s=-1/2,\ 1$

$\sin(\theta)=-\frac{1}{2}$ or $\sin(\theta)=1 \Rightarrow \theta=\frac{\pi}{2}+2\pi k,\ \frac{7\pi}{6}+2\pi k,\ \frac{11\pi}{6}+2\pi k,\ k\in\mathbb{Z}$

6e. $2c^2-7c+3=0 \Rightarrow (2c-1)(c-3)=0 \Rightarrow c=1/2,\ 3$

$\cos(\theta)=\frac{1}{2}$ or $\cos(\theta)=3 \Rightarrow \theta=\frac{\pi}{3}+2\pi k,\ \frac{5\pi}{3}+2\pi k,\ k\in\mathbb{Z}$

6f. $c\cdot s-2c=0 \Rightarrow c(s-2)=0 \Rightarrow c=0$ or $s=2$

$\cos(\theta)=0$ or $\sin(\theta)=2 \Rightarrow \theta=\frac{\pi}{2}+2\pi k,\ \frac{3\pi}{2}+2\pi k,\ k\in\mathbb{Z}$

Also acceptable: $\theta=\frac{\pi}{2}+\pi k,\ k\in\mathbb{Z}$

6g. $2c^2+s=1 \Rightarrow 2(1-s^2)+s=1 \Rightarrow 2-2s^2+s=1 \Rightarrow 2s^2-s-1=0 \Rightarrow (2s+1)(s-1)=0 \Rightarrow s=-1/2,\ 1$

$\sin(\theta)=-\frac{1}{2}$ or $\sin(\theta)=1 \Rightarrow \theta=\frac{\pi}{2}+2\pi k,\ \frac{7\pi}{6}+2\pi k,\ \frac{11\pi}{6}+2\pi k,\ k\in\mathbb{Z}$

7a. $\sqrt{2}\cos(\theta)-1=0 \Rightarrow \cos(\theta)=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2} \Rightarrow \theta=\frac{\pi}{4},\ \frac{7\pi}{4}$

7b. $4\sin^2(\theta)-3=0 \Rightarrow \sin^2(\theta)=\frac{3}{4} \Rightarrow \sin(\theta)=\pm\frac{\sqrt{3}}{2} \Rightarrow \theta=\frac{\pi}{3},\ \frac{2\pi}{3},\ \frac{4\pi}{3},\ \frac{5\pi}{3}$

7c. $\csc^2(\theta)-4=0 \Rightarrow \csc^2(\theta)=4 \Rightarrow \csc(\theta)=\pm 2 \Rightarrow \sin(\theta)=\pm\frac{1}{2} \Rightarrow \theta=\frac{\pi}{6},\ \frac{5\pi}{6},\ \frac{7\pi}{6},\ \frac{11\pi}{6}$

7d. $4c^2-4c+1=0 \Rightarrow (2c-1)(2c-1)=0 \Rightarrow c=1/2$

$\cos(\theta)=\frac{1}{2} \Rightarrow \theta=\frac{\pi}{3},\ \frac{5\pi}{3}$

7e. $2s^2+5s-12=0 \Rightarrow (2s-3)(s+4)=0 \Rightarrow s=-4,\ 3/2$

$\sin(\theta)=-4$ or $\sin(\theta)=\frac{3}{2} \Rightarrow$ no solution

7f. $ts+s=0 \Rightarrow s(t+1)=0 \Rightarrow s=0$ or $t=-1$

$\sin(\theta)=0$ or $\tan(\theta)=-1 \Rightarrow \theta=0,\ \frac{3\pi}{4},\ \pi,\ \frac{7\pi}{4}$

7g. $\csc^2(\theta)=\cot(\theta)+3 \Rightarrow 1+\cot^2(\theta)=\cot(\theta)+3 \Rightarrow \cot^2(\theta)-\cot(\theta)-2=0$

$c^2-c-2=0 \Rightarrow (c-2)(c+1)=0 \Rightarrow c=-1,\ 2$

$\cot(\theta)=-1$ or $\cot(\theta)=2 \Rightarrow \tan(\theta)=-1$ or $\tan(\theta)=\frac{1}{2} \Rightarrow \theta=\arctan\left(\frac{1}{2}\right),\ \frac{3\pi}{4},\ \arctan\left(\frac{1}{2}\right)+\pi,\ \frac{7\pi}{4}$

8a. $2\cos(2\theta)+1=0 \Rightarrow \cos(2\theta)=-\frac{1}{2} \Rightarrow 2\theta=\frac{2\pi}{3}+2\pi k,\ \frac{4\pi}{3}+2\pi k,\ k\in\mathbb{Z} \Rightarrow \theta=\frac{\pi}{3}+\pi k,\ \frac{2\pi}{3}+\pi k,\ k\in\mathbb{Z}$

8b. $\sec(4\theta)-2=0 \Rightarrow \sec(4\theta)=2 \Rightarrow \cos(4\theta)=\frac{1}{2} \Rightarrow 4\theta=\frac{\pi}{3}+2\pi k,\ \frac{5\pi}{3}+2\pi k,\ k\in\mathbb{Z} \Rightarrow \theta=\frac{\pi}{12}+\frac{\pi}{2}k,\ \frac{5\pi}{12}+\frac{\pi}{2}k,\ k\in\mathbb{Z}$

8c. $2\sin\left(\frac{\theta}{3}\right)+\sqrt{3}=0 \Rightarrow \sin\left(\frac{\theta}{3}\right)=-\frac{\sqrt{3}}{2} \Rightarrow \frac{\theta}{3}=\frac{4\pi}{3}+2\pi k,\ \frac{5\pi}{3}+2\pi k,\ k\in\mathbb{Z} \Rightarrow \theta=4\pi+6\pi k,\ 5\pi+6\pi k,\ k\in\mathbb{Z}$

9a. $0\le\theta<2\pi \Rightarrow 0\le 3\theta<6\pi$

$2\sin(3\theta)+1=0 \Rightarrow \sin(3\theta)=-\frac{1}{2} \Rightarrow 3\theta=\frac{7\pi}{6},\ \frac{11\pi}{6},\ \frac{19\pi}{6},\ \frac{23\pi}{6},\ \frac{31\pi}{6},\ \frac{35\pi}{6} \Rightarrow \theta=\frac{7\pi}{18},\ \frac{11\pi}{18},\ \frac{19\pi}{18},\ \frac{23\pi}{18},\ \frac{31\pi}{18},\ \frac{35\pi}{18}$

9b. $0\le\theta<2\pi \Rightarrow 0\le 3\theta<6\pi$

$\sqrt{3}\tan(3\theta)+1=0 \Rightarrow \tan(3\theta)=-\frac{1}{\sqrt{3}} \Rightarrow 3\theta=\frac{5\pi}{6},\ \frac{11\pi}{6},\ \frac{17\pi}{6},\ \frac{23\pi}{6},\ \frac{29\pi}{6},\ \frac{35\pi}{6} \Rightarrow \theta=\frac{5\pi}{18},\ \frac{11\pi}{18},\ \frac{17\pi}{18},\ \frac{23\pi}{18},\ \frac{29\pi}{18},\ \frac{35\pi}{18}$

9c. $0\le\theta<2\pi \Rightarrow 0\le\frac{\theta}{4}<\frac{\pi}{2}$

$\tan\left(\frac{\theta}{4}\right)+\sqrt{3}=0 \Rightarrow \tan\left(\frac{\theta}{4}\right)=-\sqrt{3} \Rightarrow$ no solution

10a. $\tan(3\theta)+1=\sec(3\theta) \Rightarrow \tan(3\theta)+1-\sec(3\theta)=0 \Rightarrow \theta=0$, 2.0943 or 2.0944, 4.1887 or 4.1888

10b. $\sin(5\theta)-\sin(3\theta)=\cos(4\theta) \Rightarrow \sin(5\theta)-\sin(3\theta)-\cos(4\theta)=0$

$\theta=0.3926$ or 0.3927, 0.5235 or 0.5236, 1.1780 or 1.1781, 1.9634 or 1.9635, 2.6179 or 2.6180, 2.7488 or 2.7489, 3.5342 or 3.5343, 4.3196 or 4.3197, 5.1050 or 5.1051, 5.8904 or 5.8905

10c. $\sin(x)=x^3 \Rightarrow \sin(x)-x^3=0 \Rightarrow x=0$, 0.9286

10d. $\frac{\cos(x)}{1+x^2}=x^2 \Rightarrow \frac{\cos(x)}{1+x^2}-x^2=0 \Rightarrow x=0.7100$

11a. $\sin(75^\circ)=\sin(30^\circ+45^\circ)=\sin(30^\circ)\cos(45^\circ)+\cos(30^\circ)\sin(45^\circ)=\frac{1}{2}\cdot\frac{\sqrt{2}}{2}+\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}=\frac{\sqrt{2}+\sqrt{6}}{4}$

11b. $\cos(195^\circ)=\cos(150^\circ+45^\circ)=\cos(150^\circ)\cos(45^\circ)-\sin(150^\circ)\sin(45^\circ)=-\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}-\frac{1}{2}\cdot\frac{\sqrt{2}}{2}=\frac{-\sqrt{6}-\sqrt{2}}{4}$

11c. $\tan(165^\circ)=\tan(120^\circ+45^\circ)=\frac{\tan(120^\circ)+\tan(45^\circ)}{1-\tan(120^\circ)\tan(45^\circ)}=\frac{-\sqrt{3}+1}{1-(-\sqrt{3})(1)}=\frac{1-\sqrt{3}}{1+\sqrt{3}}$

11d. $\sin\left(\frac{19\pi}{12}\right)=\sin\left(\frac{4\pi}{3}+\frac{\pi}{4}\right)=\sin\left(\frac{4\pi}{3}\right)\cos\left(\frac{\pi}{4}\right)+\cos\left(\frac{4\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)=-\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}-\frac{1}{2}\cdot\frac{\sqrt{2}}{2}=\frac{-\sqrt{6}-\sqrt{2}}{4}$

11e. $\cos\left(\frac{11\pi}{12}\right)=\cos\left(\frac{2\pi}{3}+\frac{\pi}{4}\right)=\cos\left(\frac{2\pi}{3}\right)\cos\left(\frac{\pi}{4}\right)-\sin\left(\frac{2\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)=-\frac{1}{2}\cdot\frac{\sqrt{2}}{2}-\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}=\frac{-\sqrt{2}-\sqrt{6}}{4}$

11f. $\tan\left(\frac{7\pi}{12}\right)=\tan\left(\frac{\pi}{3}+\frac{\pi}{4}\right)=\frac{\tan(\frac{\pi}{3})+\tan(\frac{\pi}{4})}{1-\tan(\frac{\pi}{3})\tan(\frac{\pi}{4})}=\frac{\sqrt{3}+1}{1-\sqrt{3}\cdot 1}=\frac{1+\sqrt{3}}{1-\sqrt{3}}$

12a. $\cos(x)=3/5 \Rightarrow \sin(x)=-4/5$ and $\tan(y)=-\sqrt{3} \Rightarrow y=2\pi/3$

$$\cos\left(x-\frac{2\pi}{3}\right)=\cos(x)\cos\left(\frac{2\pi}{3}\right)+\sin(x)\sin\left(\frac{2\pi}{3}\right)=\frac{3}{5}\left(-\frac{1}{2}\right)-\frac{4}{5}\cdot\frac{\sqrt{3}}{2}=\frac{-3-4\sqrt{3}}{10}$$

12b. $\sin(x)=5/13 \Rightarrow \cos(x)=12/13$ and $\cos(y)=-2/\sqrt{5} \Rightarrow \sin(y)=1/\sqrt{5}$

$$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)=\frac{5}{13}\left(-\frac{2}{\sqrt{5}}\right)+\frac{12}{13}\cdot\frac{1}{\sqrt{5}}=\frac{-10+12}{13\sqrt{5}}=\frac{2}{13\sqrt{5}}$$

12c. $\cos(x)=-1/3 \Rightarrow \tan(x)=2\sqrt{2}$ and $\sin(y)=1/4 \Rightarrow \tan(y)=-1/\sqrt{15}$

$$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}=\frac{2\sqrt{2}-\frac{1}{\sqrt{15}}}{1-2\sqrt{2}(-\frac{1}{\sqrt{15}})}\cdot\frac{\sqrt{15}}{\sqrt{15}}=\frac{2\sqrt{30}-1}{\sqrt{15}+2\sqrt{2}}$$

13a. $\sin\left(x+\frac{\pi}{6}\right)=\sin(x)\cos\left(\frac{\pi}{6}\right)+\cos(x)\sin\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}\sin(x)+\frac{1}{2}\cos(x)$

$$\sin\left(x-\frac{\pi}{6}\right)=\sin(x)\cos\left(\frac{\pi}{6}\right)-\cos(x)\sin\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}\sin(x)-\frac{1}{2}\cos(x)$$

$$\sin\left(x+\frac{\pi}{6}\right)-\sin\left(x-\frac{\pi}{6}\right)=\left[\frac{\sqrt{3}}{2}\sin(x)+\frac{1}{2}\cos(x)\right]-\left[\frac{\sqrt{3}}{2}\sin(x)-\frac{1}{2}\cos(x)\right]=\cos(x)$$

$$\sin\left(x+\frac{\pi}{6}\right)-\sin\left(x-\frac{\pi}{6}\right)=1 \Rightarrow \cos(x)=1 \Rightarrow x=0$$

13b. $\cos\left(x+\frac{\pi}{3}\right)=\cos(x)\cos\left(\frac{\pi}{3}\right)-\sin(x)\sin\left(\frac{\pi}{3}\right)=\frac{1}{2}\cos(x)-\frac{\sqrt{3}}{2}\sin(x)$

$$\cos\left(x-\frac{\pi}{3}\right)=\cos(x)\cos\left(\frac{\pi}{3}\right)+\sin(x)\sin\left(\frac{\pi}{3}\right)=\frac{1}{2}\cos(x)+\frac{\sqrt{3}}{2}\sin(x)$$

$$\left(\frac{1}{2}c-\frac{\sqrt{3}}{2}s\right)\left(\frac{1}{2}c+\frac{\sqrt{3}}{2}s\right)=\frac{1}{4}c^2-\frac{3}{4}s^2=\frac{1}{4}c^2-\frac{3}{4}(1-c^2)=\frac{1}{4}c^2-\frac{3}{4}+\frac{3}{4}c^2=c^2-\frac{3}{4}$$

$$\cos\left(x+\frac{\pi}{3}\right)\cos\left(x-\frac{\pi}{3}\right)=0 \Rightarrow \cos^2(x)-\frac{3}{4}=0 \Rightarrow \cos^2(x)=\frac{3}{4} \Rightarrow \cos(x)=\pm\frac{\sqrt{3}}{2} \Rightarrow x=\frac{\pi}{6},\ \frac{5\pi}{6},\ \frac{7\pi}{6},\ \frac{11\pi}{6}$$

14a. $\sin(x)=5/13 \Rightarrow \cos(x)=12/13,\ \tan(x)=5/12$

$$\sin(2x)=2\sin(x)\cos(x)=2\left(\frac{5}{13}\right)\left(\frac{12}{13}\right)=\frac{120}{169}$$

$$\cos(2x)=\cos^2(x)-\sin^2(x)=\left(\frac{12}{13}\right)^2-\left(\frac{5}{13}\right)^2=\frac{144}{169}-\frac{25}{169}=\frac{119}{169}$$

$$\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}=\frac{2\cdot\frac{5}{12}}{1-(\frac{5}{12})^2}=\frac{\frac{5}{6}}{1-\frac{25}{144}}\cdot\frac{144}{144}=\frac{120}{144-25}=\frac{120}{119}$$

14b. $\tan(x)=-4/3 \Rightarrow \sin(x)=4/5,\ \cos(x)=-3/5$

$$\sin(2x)=2\sin(x)\cos(x)=2\left(\frac{4}{5}\right)\left(-\frac{3}{5}\right)=-\frac{24}{25}$$

$$\cos(2x)=\cos^2(x)-\sin^2(x)=\left(-\frac{3}{5}\right)^2-\left(\frac{4}{5}\right)^2=\frac{9}{25}-\frac{16}{25}=-\frac{7}{25}$$

$$\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}=\frac{2(-\frac{4}{3})}{1-(-\frac{4}{3})^2}=\frac{-\frac{8}{3}}{1-\frac{16}{9}}\cdot\frac{9}{9}=\frac{-24}{9-16}=\frac{-24}{-7}=\frac{24}{7}$$

14c. $\sin(x) = -3/5 \Rightarrow \cos(x) = -4/5,\ \tan(x) = 3/4$

$$\sin(2x) = 2\sin(x)\cos(x) = 2\left(-\frac{3}{5}\right)\left(-\frac{4}{5}\right) = \frac{24}{25}$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) = \left(-\frac{4}{5}\right)^2 - \left(-\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

$$\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)} = \frac{2(\frac{3}{4})}{1-(\frac{3}{4})^2} = \frac{\frac{3}{2}}{1-\frac{9}{16}} \cdot \frac{16}{16} = \frac{24}{16-9} = \frac{24}{7}$$

15a. $\sin(15°) = \sin\left(\frac{30°}{2}\right) = \sqrt{\frac{1-\cos(30°)}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2} \cdot \frac{2}{2}} = \sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2}$

15b. $\cos(165°) = \cos\left(\frac{330°}{2}\right) = -\sqrt{\frac{1+\cos(330°)}{2}} = -\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2} \cdot \frac{2}{2}} = -\sqrt{\frac{2+\sqrt{3}}{4}} = -\frac{\sqrt{2+\sqrt{3}}}{2}$

15c. $\tan(22.5°) = \tan\left(\frac{45°}{2}\right) = \frac{1-\cos(45°)}{\sin(45°)} = \frac{1-\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}-1}{1} = \sqrt{2}-1$

15d. $\sin\left(\frac{9\pi}{8}\right) = \sin\left(\frac{1}{2} \cdot \frac{9\pi}{4}\right) = \sin\left(\frac{1}{2} \cdot \frac{\pi}{4}\right) = -\sqrt{\frac{1-\cos(\frac{\pi}{4})}{2}} = -\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2} \cdot \frac{2}{2}} = -\sqrt{\frac{2-\sqrt{2}}{4}} = -\frac{\sqrt{2-\sqrt{2}}}{2}$

15e. $\cos\left(\frac{3\pi}{8}\right) = \cos\left(\frac{1}{2} \cdot \frac{3\pi}{4}\right) = \sqrt{\frac{1+\cos(\frac{3\pi}{4})}{2}} = \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2} \cdot \frac{2}{2}} = \sqrt{\frac{2-\sqrt{2}}{4}} = \frac{\sqrt{2-\sqrt{2}}}{2}$

15f. $\tan\left(\frac{5\pi}{12}\right) = \tan\left(\frac{1}{2} \cdot \frac{5\pi}{6}\right) = \frac{1-\cos(\frac{5\pi}{6})}{\sin(\frac{5\pi}{6})} = \frac{1+\frac{\sqrt{3}}{2}}{\frac{1}{2}} \cdot \frac{2}{2} = \frac{2+\sqrt{3}}{1} = 2+\sqrt{3}$

16a. $\sin(x) = 3/5 \Rightarrow \cos(x) = 4/5$

$0 < x < \frac{\pi}{2} \Rightarrow 0 < \frac{x}{2} < \frac{\pi}{4} \Rightarrow \frac{x}{2}$ is in Quadrant I

$$\sin\left(\frac{x}{2}\right) = \sqrt{\frac{1-\cos(x)}{2}} = \sqrt{\frac{1-\frac{4}{5}}{2} \cdot \frac{5}{5}} = \sqrt{\frac{5-4}{10}} = \sqrt{\frac{1}{10}} = \frac{1}{\sqrt{10}}$$

$$\cos\left(\frac{x}{2}\right) = \sqrt{\frac{1+\cos(x)}{2}} = \sqrt{\frac{1+\frac{4}{5}}{2} \cdot \frac{5}{5}} = \sqrt{\frac{5+4}{10}} = \sqrt{\frac{9}{10}} = \frac{3}{\sqrt{10}}$$

$$\tan\left(\frac{x}{2}\right) = \frac{1-\cos(x)}{\sin(x)} = \frac{1-\frac{4}{5}}{\frac{3}{5}} \cdot \frac{5}{5} = \frac{5-4}{3} = \frac{1}{3}$$

16b. $\cos(x) = -4/5 \Rightarrow \sin(x) = -3/5$

$\pi < x < \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4} \Rightarrow \frac{x}{2}$ is in Quadrant II

$$\sin\left(\frac{x}{2}\right) = \sqrt{\frac{1-\cos(x)}{2}} = \sqrt{\frac{1+\frac{4}{5}}{2} \cdot \frac{5}{5}} = \sqrt{\frac{5+4}{10}} = \sqrt{\frac{9}{10}} = \frac{3}{\sqrt{10}}$$

$$\cos\left(\frac{x}{2}\right) = -\sqrt{\frac{1+\cos(x)}{2}} = -\sqrt{\frac{1-\frac{4}{5}}{2} \cdot \frac{5}{5}} = -\sqrt{\frac{5-4}{10}} = -\sqrt{\frac{1}{10}} = -\frac{1}{\sqrt{10}}$$

$$\tan\left(\frac{x}{2}\right) = \frac{1-\cos(x)}{\sin(x)} = \frac{1+\frac{4}{5}}{-\frac{3}{5}} \cdot \frac{5}{5} = \frac{5+4}{-3} = \frac{9}{-3} = -3$$

16c. $\csc(x)=3 \Rightarrow \sin(x)=1/3,\ \cos(x)=-2\sqrt{2}/3$

$\frac{\pi}{2}<x<\pi \Rightarrow \frac{\pi}{4}<\frac{x}{2}<\frac{\pi}{2} \Rightarrow \frac{x}{2}$ is in Quadrant I

$\sin\left(\frac{x}{2}\right)=\sqrt{\frac{1-\cos(x)}{2}}=\sqrt{\frac{1+\frac{2\sqrt{2}}{3}}{2}\cdot\frac{3}{3}}=\sqrt{\frac{3+2\sqrt{2}}{6}}$

$\cos\left(\frac{x}{2}\right)=\sqrt{\frac{1+\cos(x)}{2}}=\sqrt{\frac{1-\frac{2\sqrt{2}}{3}}{2}\cdot\frac{3}{3}}=\sqrt{\frac{3-2\sqrt{2}}{6}}$

$\tan(x)=\frac{1-\cos(x)}{\sin(x)}=\frac{1+\frac{2\sqrt{2}}{3}}{\frac{1}{3}}\cdot\frac{3}{3}=\frac{3+2\sqrt{2}}{1}=3+2\sqrt{2}$

17a. $\sin(2\theta)+\cos(\theta)=0 \Rightarrow 2\sin(\theta)\cos(\theta)+\cos(\theta)=0$

$2sc+c=0 \Rightarrow c(2s+1)=0 \Rightarrow c=0$ or $s=-1/2$

$\cos(x)=0$ or $\sin(x)=-1/2 \Rightarrow x=\frac{\pi}{2},\ \frac{7\pi}{6},\ \frac{3\pi}{2},\ \frac{11\pi}{6}$

17b. $\cos(2\theta)+\cos(\theta)=2 \Rightarrow 2\cos^2(\theta)-1+\cos(\theta)=2 \Rightarrow 2\cos^2(\theta)+\cos(\theta)-3=0$

$2c^2+c-3=0 \Rightarrow (2c+3)(c-1)=0 \Rightarrow c=-3/2,\ 1$

$\cos(x)=-\frac{3}{2}$ or $\cos(x)=1 \Rightarrow x=0$

17c. $\cos(2\theta)-\cos^2(\theta)=0 \Rightarrow 2\cos^2(\theta)-1-\cos^2(\theta)=0 \Rightarrow \cos^2(\theta)=1 \Rightarrow \cos(\theta)=\pm1 \Rightarrow \theta=0,\ \pi$

17d. Domain restriction: $\frac{\theta}{2}\neq\frac{\pi}{2} \Rightarrow \theta\neq\pi$

$\tan\left(\frac{\theta}{2}\right)-\sin(\theta)=0 \Rightarrow \frac{1-\cos(\theta)}{\sin(\theta)}-\sin(\theta)=0 \Rightarrow \frac{1-\cos(\theta)}{\sin(\theta)}=\sin(\theta)$

$1-\cos(\theta)=\sin^2(\theta) \Rightarrow 1-\cos(\theta)=1-\cos^2(\theta) \Rightarrow \cos^2(\theta)-\cos(\theta)=0$

$c^2-c=0 \Rightarrow c(c-1)=0 \Rightarrow c=0,\ 1$

$\cos(\theta)=0$ or $\cos(\theta)=1 \Rightarrow \theta=0,\ \frac{\pi}{2},\ \frac{3\pi}{2}$

17e. $\cos(\theta)-\sin(\theta)=\sqrt{2}\sin\left(\frac{\theta}{2}\right) \Rightarrow \cos(\theta)-\sin(\theta)=\sqrt{2}\cdot\pm\sqrt{\frac{1-\cos(\theta)}{2}} \Rightarrow \cos(\theta)-\sin(\theta)=\pm\sqrt{1-\cos(\theta)}$

$c-s=\pm\sqrt{1-c} \Rightarrow (c-s)^2=1-c \Rightarrow c^2-2cs+s^2=1-c \Rightarrow 1-2cs=1-c \Rightarrow c-2cs=0 \Rightarrow c(1-2s)=0 \Rightarrow c=0$ or $s=1/2$

$\cos(x)=0$ or $\sin(x)=1/2 \Rightarrow x=\frac{\pi}{6},\ \frac{\pi}{2},\ \frac{5\pi}{6},\ \frac{3\pi}{2}$

Check solutions: $x=\frac{\pi}{6},\ \frac{3\pi}{2}$

18a. $C=63°$, $b=638.2341$ or 638.2342, $c=570.8430$

$C=180°-A-B=180°-22°-95°=63°$

$\frac{b}{\sin(B)}=\frac{a}{\sin(A)} \Rightarrow b=\frac{a\sin(B)}{\sin(A)}=\frac{240\sin(95°)}{\sin(22°)}=638.234168$

$\frac{c}{\sin(C)}=\frac{a}{\sin(A)} \Rightarrow c=\frac{a\sin(C)}{\sin(A)}=\frac{240\sin(63°)}{\sin(22°)}=570.8430$

18b. $A=100°$, $a=89.3784$ or 89.3785, $c=70.5316$

$A=180°-A-B=180°-29°-51°=100°$

$\frac{a}{\sin(A)}=\frac{b}{\sin(B)} \Rightarrow a=\frac{b\sin(A)}{\sin(B)}=\frac{44\sin(100°)}{\sin(29°)}=89.378468$

$\frac{c}{\sin(C)}=\frac{b}{\sin(B)} \Rightarrow c=\frac{b\sin(C)}{\sin(B)}=\frac{44\sin(51°)}{\sin(29°)}=70.5316$

18c. $C = 62°$, $a = 199.5477$ or 199.5478, $b = 241.5231$ or 241.5232

$C = 180° - A - B = 180° - 50° - 68° = 62°$

$$\frac{a}{\sin(A)} = \frac{c}{\sin(C)} \Rightarrow a = \frac{c\sin(A)}{\sin(C)} = \frac{230\sin(50°)}{\sin(62°)} = 199.547768568$$

$$\frac{b}{\sin(B)} = \frac{c}{\sin(C)} \Rightarrow b = \frac{c\sin(B)}{\sin(C)} = \frac{230\sin(68°)}{\sin(62°)} = 241.52315$$

18d. $B = 85°$, $a = 5.0190$ or 5.0191, $c = 9.0976$ or 9.0977

$B = 180° - A - C = 180° - 30° - 65° = 85°$

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} \Rightarrow a = \frac{b\sin(A)}{\sin(B)} = \frac{10\sin(30°)}{\sin(85°)} = 5.019099$$

$$\frac{c}{\sin(C)} = \frac{b}{\sin(B)} \Rightarrow c = \frac{b\sin(C)}{\sin(B)} = \frac{10\sin(65°)}{\sin(85°)} = 9.097697$$

18e. Case 1: $B = 89.6381°$ or $89.6382°$, $C = 53.3618°$, $b = 49.8482$

Case 2: $B = 16.3618°$, $C = 126.6381°$ or $126.6382°$, $b = 14.0426$

$h = c\sin(A) = 40\sin(37°) = 24.0726 \Rightarrow h < a < c \Rightarrow$ two cases

$$\frac{\sin(C)}{c} = \frac{\sin(A)}{a} \Rightarrow \sin(C) = \frac{c\sin(A)}{a} \Rightarrow C = \sin^{-1}\left(\frac{c\sin(A)}{a}\right) = \sin^{-1}\left(\frac{40\sin(37°)}{30}\right) = 53.3618236772°$$

$B = 180° - A - C = 180° - 37° - C = 89.6381763228°$

$$\frac{b}{\sin(B)} = \frac{a}{\sin(A)} \Rightarrow b = \frac{a\sin(B)}{\sin(A)} = \frac{30\sin(B)}{\sin(37°)} = 49.8482$$

$C = 180° - 53.3618236772° = 126.638176323°$

$B = 180° - A - C = 180° - 37° - C = 16.3618236772°$

$$\frac{b}{\sin(B)} = \frac{a}{\sin(A)} \Rightarrow b = \frac{a\sin(B)}{\sin(A)} = \frac{30\sin(B)}{\sin(37°)} = 14.0426$$

18f. Case 1: $A = 100.7277°$ or $100.7278°$, $B = 41.2722°$, $a = 67.0270$

Case 2: $A = 3.2722°$, $B = 138.7277°$ or $138.7278°$, $a = 3.8939$ or 3.8940

$h = b\sin(C) = 45\sin(38°) = 27.704766 \Rightarrow h < c < b \Rightarrow$ two cases

$$\frac{\sin(B)}{b} = \frac{\sin(C)}{c} \Rightarrow \sin(B) = \frac{b\sin(C)}{c} \Rightarrow B = \sin^{-1}\left(\frac{b\sin(C)}{c}\right) = \sin^{-1}\left(\frac{45\sin(38°)}{42}\right) = 41.2722167114°$$

$A = 180° - B - C = 180° - B - 38° = 100.727783289°$

$$\frac{a}{\sin(A)} = \frac{c}{\sin(C)} \Rightarrow a = \frac{c\sin(A)}{\sin(C)} = \frac{42\sin(A)}{\sin(38°)} = 67.0270$$

$B = 180° - 41.2722167114° = 138.727783289°$

$A = 180° - B - C = 180° - B - 38° = 3.27221671142°$

$$\frac{a}{\sin(A)} = \frac{c}{\sin(C)} \Rightarrow a = \frac{c\sin(A)}{\sin(C)} = \frac{42\sin(A)}{\sin(38°)} = 3.89395$$

18g. $B = 37.7948°$, $C = 12.2051°$ or $12.2052°$, $c = 27.5980$

$a > b \Rightarrow$ one case

$$\frac{\sin(B)}{b} = \frac{\sin(A)}{a} \Rightarrow \sin(B) = \frac{b\sin(A)}{a} \Rightarrow B = \sin^{-1}\left(\frac{b\sin(A)}{a}\right) = \sin^{-1}\left(\frac{80\sin(130°)}{100}\right) = 37.7948156296°$$

$C = 180° - A - B = 180° - 130° - B = 12.2051843704°$

$$\frac{c}{\sin(C)} = \frac{a}{\sin(A)} \Rightarrow c = \frac{a\sin(C)}{\sin(A)} = \frac{100\sin(C)}{\sin(130°)} = 27.5980$$

19. $a \ge b \Rightarrow$ one case

$$\frac{\sin(B)}{b} = \frac{\sin(A)}{a} \Rightarrow \sin(B) = \frac{b\sin(A)}{a} \Rightarrow B = \sin^{-1}\left(\frac{b\sin(A)}{a}\right) = \sin^{-1}\left(\frac{312\sin(48.6°)}{527}\right) = 26.3650377232°$$

$$C = 180° - A - B = 180° - 48.6° - B = 105.034962277°$$

$$\frac{c}{\sin(C)} = \frac{a}{\sin(A)} \Rightarrow c = \frac{a\sin(C)}{\sin(A)} = \frac{527\sin(C)}{\sin(48.6°)} = 678.5122 \text{ or } 678.5123 \text{ feet}$$

20a. $A = 49.8683°$ or $49.8684°$, $B = 72.8833°$, $C = 57.2482°$ or $57.2483°$

$$\cos(A) = \frac{b^2+c^2-a^2}{2bc} \Rightarrow A = \cos^{-1}\left(\frac{b^2+c^2-a^2}{2bc}\right) = \cos^{-1}\left(\frac{25^2+22^2-20^2}{2\cdot 25\cdot 22}\right) = 49.8683962361°$$

$$\cos(B) = \frac{a^2+c^2-b^2}{2ac} \Rightarrow B = \cos^{-1}\left(\frac{a^2+c^2-b^2}{2ac}\right) = \cos^{-1}\left(\frac{20^2+22^2-25^2}{2\cdot 20\cdot 22}\right) = 72.8833429188°$$

$$C = 180° - A - B = 57.2482608451°$$

20b. $A = 38.6248°$, $B = 48.5091°$ or $48.5092°$, $C = 92.8659°$ or $92.8660°$

$$\cos(A) = \frac{b^2+c^2-a^2}{2bc} \Rightarrow A = \cos^{-1}\left(\frac{b^2+c^2-a^2}{2bc}\right) = \cos^{-1}\left(\frac{12^2+16^2-10^2}{2\cdot 12\cdot 16}\right) = 38.6248328731°$$

$$\cos(B) = \frac{a^2+c^2-b^2}{2ac} \Rightarrow B = \cos^{-1}\left(\frac{a^2+c^2-b^2}{2ac}\right) = \cos^{-1}\left(\frac{10^2+16^2-12^2}{2\cdot 10\cdot 16}\right) = 48.5091831443°$$

$$C = 180° - A - B = 92.8659839826°$$

20c. $A = 47.6984°$, $B = 21.2761°$, $C = 111.0254°$ or $111.0255°$

$$\cos(A) = \frac{b^2+c^2-a^2}{2bc} \Rightarrow A = \cos^{-1}\left(\frac{b^2+c^2-a^2}{2bc}\right) = \cos^{-1}\left(\frac{60.1^2+154.6^2-122.5^2}{2\cdot 60.1\cdot 154.6}\right) = 47.6984063018°$$

$$\cos(B) = \frac{a^2+c^2-b^2}{2ac} \Rightarrow B = \cos^{-1}\left(\frac{a^2+c^2-b^2}{2ac}\right) = \cos^{-1}\left(\frac{122.5^2+154.6^2-60.1^2}{2\cdot 122.5\cdot 154.6}\right) = 21.2761201925°$$

$$C = 180° - A - B = 111.025473506°$$

20d. $A = 47.5115°$ or $47.5116°$, $B = 79.4884°$, $c = 3.2490$ or 3.2491

$$c^2 = a^2 + b^2 - 2ab\cos(C) \Rightarrow c = \sqrt{3^2+4^2-2\cdot 3\cdot 4\cdot\cos(53°)} = 3.24906747304$$

$$\cos(A) = \frac{b^2+c^2-a^2}{2bc} \Rightarrow A = \cos^{-1}\left(\frac{b^2+c^2-a^2}{2bc}\right) = \cos^{-1}\left(\frac{4^2+c^2-3^2}{2\cdot 4\cdot c}\right) = 47.5115572385°$$

$$B = 180° - A - C = 180° - A - 53° = 79.4884427615°$$

20e. $B = 80.4567°$ or $80.4568°$, $C = 29.5432°$, $a = 57.1727$ or 57.1728

$$a^2 = b^2 + c^2 - 2bc\cos(A) \Rightarrow a = \sqrt{60^2+30^2-2\cdot 60\cdot 30\cdot\cos(70°)} = 57.1727862189$$

$$\cos(B) = \frac{a^2+c^2-b^2}{2ac} \Rightarrow B = \cos^{-1}\left(\frac{a^2+c^2-b^2}{2ac}\right) = \cos^{-1}\left(\frac{a^2+30^2-60^2}{2\cdot a\cdot 30}\right) = 80.4567527197°$$

$$C = 180° - A - B = 29.5432472803°$$

20f. $A = 78.9722°$ or $78.9723°$, $C = 49.0277°$, $b = 52.1842$ or 52.1843

$$b^2 = a^2 + c^2 - 2ac\cos(B) \Rightarrow b = \sqrt{65^2+50^2-2\cdot 65\cdot 50\cdot\cos(52°)} = 52.1842927554$$

$$\cos(A) = \frac{b^2+c^2-a^2}{2bc} \Rightarrow A = \cos^{-1}\left(\frac{b^2+c^2-a^2}{2bc}\right) = \cos^{-1}\left(\frac{b^2+50^2-65^2}{2\cdot b\cdot 50}\right) = 78.9722916129°$$

$$C = 180° - A - B = 180° - A - 52° = 49.0277083871°$$

21. $x^2 = 15^2 + 25^2 - 2\cdot 15\cdot 25\cdot\cos(65°) \Rightarrow x = \sqrt{15^2+25^2-2\cdot 15\cdot 25\cdot\cos(65°)} = 23.0875$ or 23.0876 miles

22. $\mathbf{v} = \langle 5-3,\ 4-(-1)\rangle = \langle 2,\ 5\rangle$

$= (5-3)\mathbf{i} + (4-(-1))\mathbf{j} = 2\mathbf{i} + 5\mathbf{j}$

$\|\mathbf{v}\| = \sqrt{2^2+5^2} = \sqrt{4+25} = \sqrt{29}$

$\mathbf{v} = \langle -2-(-3),\ 9-(-7)\rangle = \langle 1,\ 16\rangle$

$= (-2-(-3))\mathbf{i} + (9-(-7))\mathbf{j} = \mathbf{i} + 16\mathbf{j}$

$\|\mathbf{v}\| = \sqrt{1^2+16^2} = \sqrt{1+256} = \sqrt{257}$

23a. $3\mathbf{u}-5\mathbf{v}=3\langle 3,\ -2\rangle-5\langle 2,\ -4\rangle=\langle 9,\ -6\rangle-\langle 10,\ 20\rangle=\langle 9-10,\ -6+20\rangle=\langle -1,\ 14\rangle$

23b. $3\mathbf{u}-5\mathbf{v}=3(4\mathbf{i}-3\mathbf{j})-5(-2\mathbf{i}+3\mathbf{j})=12\mathbf{i}-9\mathbf{j}+10\mathbf{i}-15\mathbf{j}=22\mathbf{i}-24\mathbf{j}$

24a. $\|\mathbf{v}\|=\sqrt{(-12)^2+(-5)^2}=\sqrt{144+25}=\sqrt{169}=13\Rightarrow \mathbf{u}=\dfrac{\mathbf{v}}{\|\mathbf{v}\|}=\dfrac{1}{13}\langle -12,\ -5\rangle=\left\langle -\dfrac{12}{13},\ -\dfrac{5}{13}\right\rangle$

24b. $\|\mathbf{v}\|=\sqrt{3^2+(-5)^2}=\sqrt{9+25}=\sqrt{34}\Rightarrow \mathbf{u}=\dfrac{\mathbf{v}}{\|\mathbf{v}\|}=\dfrac{1}{\sqrt{34}}\langle 3,\ -5\rangle=\left\langle \dfrac{3}{\sqrt{34}},\ -\dfrac{5}{\sqrt{34}}\right\rangle$

25a. $\tan(\theta)=\dfrac{-2}{2\sqrt{3}}=-\dfrac{1}{\sqrt{3}}\Rightarrow \theta=\dfrac{11\pi}{6}$

25b. $\tan(\theta)=\dfrac{-3}{-3}=1\Rightarrow \theta=\dfrac{5\pi}{4}$

25c. $\tan(\theta)=\dfrac{5}{2}\Rightarrow \theta=\arctan\left(\dfrac{5}{2}\right)=1.1902$ or 1.1903 radians $=68.1985^\circ$ or 68.1986°

26a. $\mathbf{u}\cdot\mathbf{v}=(-3)(-2)+7(-5)=6-35=-29$

26b. $\mathbf{u}\cdot\mathbf{v}=(-2)(-3)+(-5)(1)=6-5=1$

27a. $\mathbf{u}\cdot\mathbf{u}=3(3)+(-7)(-7)=9+49=58\Rightarrow \|\mathbf{u}\|=\sqrt{58}$

27b. $\mathbf{u}\cdot\mathbf{u}=(-4)(-4)+(-6)(-6)=16+36=52\Rightarrow \|\mathbf{u}\|=\sqrt{52}=\sqrt{4}\cdot\sqrt{13}=2\sqrt{13}$

28a. $\mathbf{u}\cdot\mathbf{v}=4(-6)+(-3)(-9)=-24+27=3,\quad \|\mathbf{u}\|=\sqrt{4^2+(-3)^2}=5$ and $\|\mathbf{v}\|=\sqrt{(-6)^2+(-9)^2}=\sqrt{36+81}=\sqrt{117}=\sqrt{9}\cdot\sqrt{13}=3\sqrt{13}$

$\theta=\cos^{-1}\left(\dfrac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|}\right)=\cos^{-1}\left(\dfrac{3}{5\cdot 3\sqrt{13}}\right)=\cos^{-1}\left(\dfrac{1}{5\sqrt{13}}\right)=86.8201^\circ$ or 86.8202°

28b. $\mathbf{u}\cdot\mathbf{v}=(-2)(4)+7(3)=-8+21=13,\quad \|\mathbf{u}\|=\sqrt{(-2)^2+7^2}=\sqrt{4+49}=\sqrt{53}$ and $\|\mathbf{v}\|=\sqrt{4^2+3^2}=5$

$\theta=\cos^{-1}\left(\dfrac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|}\right)=\cos^{-1}\left(\dfrac{13}{5\sqrt{53}}\right)=69.0754^\circ$ or 69.0755°

29. $\mathbf{u}\cdot\mathbf{v}=3k(2)+5(7)=6k+35$

$\mathbf{u}\cdot\mathbf{v}=0\Rightarrow 6k+35=0\Rightarrow k=-35/6$

$\mathbf{u}\cdot\mathbf{v}=(-4)(7)+6(2k)=-28+12k$

$\mathbf{u}\cdot\mathbf{v}=0\Rightarrow -28+12k=0\Rightarrow k=28/12\Rightarrow k=7/3$

30a. $\sqrt{(-3)^2+(-3)^2}=\sqrt{9+9}=\sqrt{18}=\sqrt{9}\cdot\sqrt{2}=3\sqrt{2}$ and $\tan(\theta)=\dfrac{-3}{-3}=1\Rightarrow \theta=\dfrac{5\pi}{4}$

$-3-3i=3\sqrt{2}\ \text{cis}\left(\dfrac{5\pi}{4}\right)$

30b. $\sqrt{(-2)^2+(2\sqrt{3})^2}=\sqrt{4+12}=\sqrt{16}=4$ and $\tan(\theta)=\dfrac{2\sqrt{3}}{-2}=-\sqrt{3}\Rightarrow \theta=\dfrac{2\pi}{3}$

$-2+2\sqrt{3}\,i=4\,\text{cis}\left(\dfrac{2\pi}{3}\right)$

30c. $\theta=\pi\Rightarrow -4=4\,\text{cis}(\pi)$

30d. $\theta=\dfrac{3\pi}{2}\Rightarrow -2i=2\,\text{cis}\left(\dfrac{3\pi}{2}\right)$

31a. $4\,\text{cis}(210°) = 4\left(\cos(210°) + i\sin(210°)\right) = 4\left(-\dfrac{\sqrt{3}}{2} - \dfrac{1}{2}i\right) = -2\sqrt{3} - 2i$

31b. $-6\,\text{cis}\left(\dfrac{5\pi}{3}\right) = -6\left(\cos\left(\dfrac{5\pi}{3}\right) + i\sin\left(\dfrac{5\pi}{3}\right)\right) = -6\left(\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i\right) = -3 + 3\sqrt{3}\,i$

32a. $\dfrac{5}{2}\text{cis}\left(\dfrac{\pi}{3}\right)\cdot 4\,\text{cis}\left(\dfrac{5\pi}{6}\right) = \dfrac{5}{2}\cdot 4\cdot \text{cis}\left(\dfrac{\pi}{3} + \dfrac{5\pi}{6}\right) = 10\,\text{cis}\left(\dfrac{7\pi}{6}\right) = 10\left(\cos\left(\dfrac{7\pi}{6}\right) + i\sin\left(\dfrac{7\pi}{6}\right)\right) = 10\left(-\dfrac{\sqrt{3}}{2} - \dfrac{1}{2}i\right) = -5\sqrt{3} - 5i$

32b. $\dfrac{12\,\text{cis}(2\pi/3)}{3\,\text{cis}(\pi/6)} = \dfrac{12}{3}\text{cis}\left(\dfrac{2\pi}{3} - \dfrac{\pi}{6}\right) = 4\,\text{cis}\left(\dfrac{\pi}{2}\right) = 4\left(\cos\left(\dfrac{\pi}{2}\right) + i\sin\left(\dfrac{\pi}{2}\right)\right) = 4(0 + i) = 4i$

33a. $\sqrt{(-4)^2 + 4^2} = \sqrt{16+16} = \sqrt{32} = \sqrt{16}\cdot\sqrt{2} = 4\sqrt{2}$ and $\tan(\theta_1) = \dfrac{4}{-4} = -1 \Rightarrow \theta_1 = \dfrac{3\pi}{4}$

$\sqrt{(-2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = \sqrt{4}\cdot\sqrt{2} = 2\sqrt{2}$ and $\tan(\theta_2) = \dfrac{-2}{-2} = 1 \Rightarrow \theta_2 = \dfrac{5\pi}{4}$

$(-4+4i)(-2-2i) = 4\sqrt{2}\,\text{cis}\left(\dfrac{3\pi}{4}\right)\cdot 2\sqrt{2}\,\text{cis}\left(\dfrac{5\pi}{4}\right) = 4\sqrt{2}\cdot 2\sqrt{2}\,\text{cis}\left(\dfrac{3\pi}{4} + \dfrac{5\pi}{4}\right) = 16\,\text{cis}(2\pi) = 16\,\text{cis}(0)$

$= 16\left(\cos(0) + i\sin(0)\right) = 16(1 + i\cdot 0) = 16$

33b. $\theta_1 = 3\pi/2$

$\sqrt{3^2 + (3\sqrt{3})^2} = \sqrt{9+27} = \sqrt{36} = 6$ and $\tan(\theta_2) = \dfrac{3\sqrt{3}}{3} = \sqrt{3} \Rightarrow \theta_2 = \dfrac{\pi}{3}$

$\dfrac{-4i}{3+3\sqrt{3}\,i} = \dfrac{4\,\text{cis}(3\pi/2)}{6\,\text{cis}(\pi/3)} = \dfrac{4}{6}\text{cis}\left(\dfrac{3\pi}{2} - \dfrac{\pi}{3}\right) = \dfrac{2}{3}\text{cis}\left(\dfrac{7\pi}{6}\right) = \dfrac{2}{3}\left(\cos\left(\dfrac{7\pi}{6}\right) + i\sin\left(\dfrac{7\pi}{6}\right)\right) = \dfrac{2}{3}\left(-\dfrac{\sqrt{3}}{2} - \dfrac{1}{2}i\right) = -\dfrac{\sqrt{3}}{3} - \dfrac{1}{3}i$

34a. $\sqrt{5^2 + (-5)^2} = \sqrt{25+25} = \sqrt{50} = \sqrt{25}\cdot\sqrt{2} = 5\sqrt{2}$ and $\tan(\theta) = \dfrac{-5}{5} = -1 \Rightarrow \theta = \dfrac{7\pi}{4}$

$(5-5i)^3 = \left(5\sqrt{2}\,\text{cis}\left(\dfrac{7\pi}{4}\right)\right)^3 = (5\sqrt{2})^3\,\text{cis}\left(3\cdot\dfrac{7\pi}{4}\right) = 250\sqrt{2}\,\text{cis}\left(\dfrac{21\pi}{4}\right) = 250\sqrt{2}\,\text{cis}\left(\dfrac{5\pi}{4}\right)$

$= 250\sqrt{2}\left(\cos\left(\dfrac{5\pi}{4}\right) + i\sin\left(\dfrac{5\pi}{4}\right)\right) = 250\sqrt{2}\left(-\dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{2}}{2}i\right) = -250 - 250i$

34b. $\left(4\,\text{cis}(75°)\right)^2 = 4^2\,\text{cis}(2\cdot 75°) = 16\,\text{cis}(150°) = 16\left(\cos(150°) + i\sin(150°)\right) = 16\left(-\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}i\right) = -8\sqrt{3} + 8i$

34c. $\left(2\,\text{cis}\left(\dfrac{\pi}{18}\right)\right)^6 = 2^6\,\text{cis}\left(6\cdot\dfrac{\pi}{18}\right) = 64\,\text{cis}\left(\dfrac{\pi}{3}\right) = 64\left(\cos\left(\dfrac{\pi}{3}\right) + i\sin\left(\dfrac{\pi}{3}\right)\right) = 64\left(\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i\right) = 32 + 32\sqrt{3}\,i$

35a. $\theta = \pi/2$

$$\sqrt{9i} = \left(9\,\text{cis}\left(\frac{\pi}{2}\right)\right)^{1/2} = 9^{1/2}\,\text{cis}\left(\frac{\frac{\pi}{2}+2\pi k}{2}\right),\ k=0,\,1 \Rightarrow \sqrt{9i} = 3\,\text{cis}\left(\frac{\pi}{4}\right),\ 3\,\text{cis}\left(\frac{5\pi}{4}\right)$$

$$3\,\text{cis}\left(\frac{\pi}{4}\right) = 3\left(\cos\left(\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{4}\right)\right) = 3\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right) = \frac{3\sqrt{2}}{2}+\frac{3\sqrt{2}}{2}i$$

$$3\,\text{cis}\left(\frac{5\pi}{4}\right) = 3\left(\cos\left(\frac{5\pi}{4}\right)+i\sin\left(\frac{5\pi}{4}\right)\right) = 3\left(-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i\right) = -\frac{3\sqrt{2}}{2}-\frac{3\sqrt{2}}{2}i$$

35b. $\theta = 3\pi/2$

$$\sqrt[3]{-8i} = \left(8\,\text{cis}\left(\frac{3\pi}{2}\right)\right)^{1/3} = 8^{1/3}\,\text{cis}\left(\frac{\frac{3\pi}{2}+2\pi k}{3}\right),\ k=0,\,1,\,2 \Rightarrow \sqrt[3]{-8i} == 2\,\text{cis}\left(\frac{\pi}{2}\right),\ 2\,\text{cis}\left(\frac{7\pi}{6}\right),\ 2\,\text{cis}\left(\frac{11\pi}{6}\right)$$

$$2\,\text{cis}\left(\frac{\pi}{2}\right) = 2\left(\cos\left(\frac{\pi}{2}\right)+i\sin\left(\frac{\pi}{2}\right)\right) = 2(0+i) = 2i$$

$$2\,\text{cis}\left(\frac{7\pi}{6}\right) = 2\left(\cos\left(\frac{7\pi}{6}\right)+i\sin\left(\frac{7\pi}{6}\right)\right) = 2\left(-\frac{\sqrt{3}}{2}-\frac{1}{2}i\right) = -\sqrt{3}-i$$

$$2\,\text{cis}\left(\frac{11\pi}{6}\right) = 2\left(\cos\left(\frac{11\pi}{6}\right)+i\sin\left(\frac{11\pi}{6}\right)\right) = 2\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right) = \sqrt{3}-i$$

35c. $\sqrt{(-8)^2+(8\sqrt{3})^2} = \sqrt{64+192} = \sqrt{256} = 16$ and $\tan(\theta) = \frac{16\sqrt{3}}{-16} = -\sqrt{3} \Rightarrow \theta = \frac{2\pi}{3}$

$$\sqrt[4]{-8+8\sqrt{3}} = \left(16\,\text{cis}\left(\frac{2\pi}{3}\right)\right)^{1/4} = 16^{1/4}\,\text{cis}\left(\frac{\frac{2\pi}{3}+2\pi k}{4}\right),\ k=0,\,1,\,2,\,3 \Rightarrow \sqrt[4]{-8+8\sqrt{3}} = 2\,\text{cis}\left(\frac{\pi}{6}\right),\ 2\,\text{cis}\left(\frac{2\pi}{3}\right),\ 2\,\text{cis}\left(\frac{7\pi}{6}\right),\ 2\,\text{cis}\left(\frac{5\pi}{3}\right)$$

$$2\,\text{cis}\left(\frac{\pi}{6}\right) = 2\left(\cos\left(\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{6}\right)\right) = 2\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right) = \sqrt{3}+i$$

$$2\,\text{cis}\left(\frac{2\pi}{3}\right) = 2\left(\cos\left(\frac{2\pi}{3}\right)+i\sin\left(\frac{2\pi}{3}\right)\right) = 2\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right) = -1+\sqrt{3}\,i$$

$$2\,\text{cis}\left(\frac{7\pi}{6}\right) = 2\left(\cos\left(\frac{7\pi}{6}\right)+i\sin\left(\frac{7\pi}{6}\right)\right) = 2\left(-\frac{\sqrt{3}}{2}-\frac{1}{2}i\right) = -\sqrt{3}-i$$

$$2\,\text{cis}\left(\frac{5\pi}{3}\right) = 2\left(\cos\left(\frac{5\pi}{3}\right)+i\sin\left(\frac{5\pi}{3}\right)\right) = 2\left(\frac{1}{2}-\frac{\sqrt{3}}{2}i\right) = 1-\sqrt{3}\,i$$