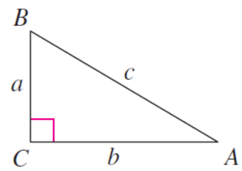
Trigonometry/Precalculus — Name ______________________

Quarter 2 Exam Study Guide — Date ______________ Pd ______

§4.8 Applications and Models

1. Solve the right triangle shown in the figure.
   a. $A = 6.8°,\ a = 21.7$
   b. $A = 28° \quad b = 9$
   c. $a = 24,\ c = 49$



2. From the top of a 200-ft lighthouse, the angle of depression to a ship in the ocean is $23°$. How far is the ship from the base of the lighthouse?

3. A man is lying on the beach, flying a kite. He holds the end of the kite string at ground level and estimates the angle of elevation of the kite to be $50°$. If the string is 450 feet long, how high is the kite above the ground?

4. To estimate the height of a mountain above a level plain, the angle of elevation to the top of the mountain is measured to be $32°$. One thousand feet closer to the mountain along the plain, it is found that the angle of elevation is $35°$. Estimate the height of the mountain.

§5.1 and §5.2 Verifying Fundamental Identities

5. Verify the identity.
   a. $\dfrac{\sin(\theta)}{\tan(\theta)} = \cos(\theta)$
   b. $\tan(\theta) + \cot(\theta) = \sec(\theta)\csc(\theta)$
   c. $\left(\sin(x) + \cos(x)\right)^2 = 1 + 2\sin(x)\cos(x)$
   d. $(1 - \cos(\beta))(1 + \cos(\beta)) = \dfrac{1}{\csc^2(\beta)}$
   e. $\left(\tan(x) + \cot(x)\right)^2 = \sec^2(x) + \csc^2(x)$
   f. $\dfrac{\sec(x) + \csc(x)}{\tan(x) + \cot(x)} = \sin(x) + \cos(x)$
   g. $\dfrac{\sin^3(x) + \cos^3(x)}{\sin(x) + \cos(x)} = 1 - \sin(x)\cos(x)$
   h. $\dfrac{\cos(\theta)}{1 - \sin(\theta)} = \sec(\theta) + \tan(\theta)$

§5.3a Solving Trigonometric Equations

6. Solve the equation algebraically.
   a. $\sqrt{2}\sin(\theta) + 1 = 0$
   b. $3\tan^2(\theta) - 1 = 0$
   c. $\sec^2(\theta) - 2 = 0$
   d. $2\sin^2(\theta) - \sin(\theta) - 1 = 0$
   e. $2\cos^2(\theta) - 7\cos(\theta) + 3 = 0$
   f. $\cos(\theta)\sin(\theta) - 2\cos(\theta) = 0$
   g. $2\cos^2(\theta) + \sin(\theta) = 1$

7. Find all solutions of the equation in the interval $[0, 2\pi)$ algebraically.
   a. $\sqrt{2}\cos(\theta) - 1 = 0$
   b. $4\sin^2(\theta) - 3 = 0$
   c. $\csc^2(\theta) - 4 = 0$
   d. $4\cos^2(\theta) - 4\cos(\theta) + 1 = 0$
   e. $2\sin^2(\theta) + 5\sin(\theta) - 12 = 0$
   f. $\tan(\theta)\sin(\theta) + \sin(\theta) = 0$
   g. $\csc^2(\theta) = \cot(\theta) + 3$

§5.3b Solving Trigonometric Equations

8. Solve the equation algebraically.
   a. $2\cos(2\theta) + 1 = 0$
   b. $\sec(4\theta) - 2 = 0$
   c. $2\sin\left(\dfrac{\theta}{3}\right) + \sqrt{3} = 0$

9. Find all solutions of the equation in the interval $[0, 2\pi)$ algebraically.
   a. $2\sin(3\theta) + 1 = 0$
   b. $\sqrt{3}\tan(3\theta) + 1 = 0$
   c. $\tan\left(\dfrac{\theta}{4}\right) + \sqrt{3} = 0$

10. Find all solutions of the equation in the interval $[0, 2\pi)$ graphically.
   a. $\tan(3\theta) + 1 = \sec(3\theta)$
   b. $\sin(5\theta) - \sin(3\theta) = \cos(4\theta)$
   c. $\sin(x) = x^3$
   d. $\dfrac{\cos(x)}{1 + x^2} = x^2$

§5.4 Sum and Difference Formulas

11. Find the exact value of the expression.
    a. $\sin(75°)$
    b. $\cos(195°)$
    c. $\tan(165°)$
    d. $\sin\left(\frac{19\pi}{12}\right)$
    e. $\cos\left(\frac{11\pi}{12}\right)$
    f. $\tan\left(\frac{7\pi}{12}\right)$

12. Evaluate each expression under the given conditions.
    a. $\cos(x-y)$, $\cos(x)=3/5$, x in Quadrant IV, $\tan(y)=-\sqrt{3}$, y in Quadrant II
    b. $\sin(x+y)$, $\sin(x)=5/13$, x in Quadrant I, $\cos(y)=-2/\sqrt{5}$, y in Quadrant II
    c. $\tan(x+y)$, $\cos(x)=-1/3$, x in Quadrant III, $\sin(y)=1/4$, y in Quadrant II

13. Find all solutions of the equation in the interval $[0, 2\pi)$ algebraically.
    a. $\sin\left(x+\frac{\pi}{6}\right)-\sin\left(x-\frac{\pi}{6}\right)=1$
    b. $\cos\left(x+\frac{\pi}{3}\right)\cos\left(x-\frac{\pi}{3}\right)=0$

§5.5 Multiple-Angle Formulas

14. Find $\sin(2x)$, $\cos(2x)$, and $\tan(2x)$ from the given information.
    a. $\sin(x)=5/13$, x in Quadrant I
    b. $\tan(x)=-4/3$, x in Quadrant II
    c. $\sin(x)=-3/5$, x in Quadrant III

15. Find the exact value of the expression.
    a. $\sin(15°)$
    b. $\cos(165°)$
    c. $\tan(22.5°)$
    d. $\sin\left(\frac{9\pi}{8}\right)$
    e. $\cos\left(\frac{3\pi}{8}\right)$
    f. $\tan\left(\frac{5\pi}{12}\right)$

16. Find $\sin(x/2)$, $\cos(x/2)$, and $\tan(x/2)$ from the given information.
    a. $\sin(x)=\frac{3}{5}$, x in Quadrant I
    b. $\cos(x)=-4/5$, x in Quadrant III
    c. $\csc(x)=3$, x in Quadrant II

17. Find all solutions of the equation in the interval $[0, 2\pi)$ algebraically.
    a. $\sin(2\theta)+\cos(\theta)=0$
    b. $\cos(2\theta)+\cos(\theta)=2$
    c. $\cos(2\theta)-\cos^2(\theta)=0$
    d. $\tan\left(\frac{\theta}{2}\right)-\sin(\theta)=0$
    e. $\cos(\theta)-\sin(\theta)=\sqrt{2}\sin\left(\frac{\theta}{2}\right)$

§6.1 Law of Sines

18. Use the Law of Sines to solve the triangle. If two solutions exist, find both.
    a. $A=22°$, $B=95°$, $a=240$
    b. $B=29°$, $C=51°$, $b=44$
    c. $A=50°$, $B=68°$, $c=230$
    d. $A=30°$, $C=65°$, $b=10$
    e. $a=30$, $c=40$, $A=37°$
    f. $b=45$, $c=42$, $C=38°$
    g. $a=100$, $b=80$, $A=130°$

19. Points A and B are separated by a lake. To find the distance between them, a surveyor locates a point C on land such that $\angle CAB=48.6°$. He also measures CA as 312 feet and CB as 527 feet. Find the distance between A and B.

§6.2 Law of Cosines

20. Use the Law of Cosines to solve the triangle.
    a. $a=20$, $b=25$, $c=22$
    b. $a=10$, $b=12$, $c=16$
    c. $a=122.5$, $b=60.1$, $c=154.6$
    d. $a=3$, $b=4$, $C=53°$
    e. $b=60$, $c=30$, $A=70°$
    f. $a=65$, $c=50$, $B=52°$

21. Two straight roads diverge at an angle of $65°$. Two cars leave the intersection at 2:00 p.m., one traveling at 50 mph and the other at 30 mph. How far apart are the cars at 2:30 p.m.?

§6.3 Vectors in the Plane

22. The initial and terminal points of a vector are given. Write the vector **v** in both component form and as a linear combination of the standard unit vectors **i** and **j**. Then find the magnitude of **v**.
    a. initial point $(3,\ -1)$ and terminal point $(5,\ 4)$
    b. initial point $(-3,\ -7)$ and terminal point $(-2,\ 9)$

23. Find $3\mathbf{u}-5\mathbf{v}$.
    a. $\mathbf{u}=\langle 3,\ -2\rangle$ and $\mathbf{v}=\langle 2,\ -4\rangle$
    b. $\mathbf{u}=4\mathbf{i}-3\mathbf{j}$ and $\mathbf{v}=-2\mathbf{i}+3\mathbf{j}$

24. Find a unit vector in the direction of the given vector.
    a. $\mathbf{v}=\langle -12,\ -5\rangle$
    b. $\mathbf{v}=\langle 3,\ -5\rangle$

25. Find the magnitude and direction angle of the vector **v**.
    a. $\mathbf{v}=2\sqrt{3}\,\mathbf{i}-2\mathbf{j}$
    b. $\mathbf{v}=-3\mathbf{i}-3\mathbf{j}$
    c. $\mathbf{v}=2\mathbf{i}+5\mathbf{j}$

§6.4 Vectors and Dot Products

26. Find the dot product of **u** and **v**.
    a. $\mathbf{u}=\langle -3,\ 7\rangle,\ \ \mathbf{v}=\langle -2,\ -5\rangle$
    b. $\mathbf{u}=-2\mathbf{i}-5\mathbf{j},\ \ \mathbf{v}=-3\mathbf{i}+\mathbf{j}$

27. Use the dot product to find the magnitude of **u**.
    a. $\mathbf{u}=\langle 3,\ -7\rangle$
    b. $\mathbf{u}=-4\mathbf{i}-6\mathbf{j}$

28. Find the angle θ between the vectors.
    a. $\mathbf{u}=4\mathbf{i}-3\mathbf{j},\ \ \mathbf{v}=-6\mathbf{i}-9\mathbf{j}$
    b. $\mathbf{u}=-2\mathbf{i}+7\mathbf{j},\ \ \mathbf{v}=4\mathbf{i}+3\mathbf{j}$

29. Find the value of k so that the vectors **u** and **v** are orthogonal.
    a. $\mathbf{u}=3k\mathbf{i}+5\mathbf{j},\ \ \mathbf{v}=2\mathbf{i}+7\mathbf{j}$
    b. $\mathbf{u}=-4\mathbf{i}+6\mathbf{j},\ \ \mathbf{v}=7\mathbf{i}+2k\mathbf{j}$

§6.5a Trigonometric Form of a Complex Number

30. Find the trigonometric form of the number.
    a. $-3-3i$
    b. $-2+2\sqrt{3}\,i$
    c. -4
    d. $-2i$

31. Find the standard form of the number.
    a. $4\,\text{cis}(210°)$
    b. $-6\,\text{cis}\left(\frac{5\pi}{3}\right)$

32. Evaluate.
    a. $\frac{5}{2}\text{cis}\left(\frac{\pi}{3}\right)\cdot 4\,\text{cis}\left(\frac{5\pi}{6}\right)$
    b. $\dfrac{12\,\text{cis}(2\pi/3)}{3\,\text{cis}(\pi/6)}$

33. Rewrite each complex number in trigonometric form, then evaluate.
    a. $(-4+4i)(-2-2i)$
    b. $\dfrac{-4i}{3+3\sqrt{3}\,i}$

§6.5b Powers and Roots of Complex Numbers

34. Use DeMoivre's Theorem to find the indicated power of the complex number. Write your answer in standard form.
    a. $(5-5i)^3$
    b. $\left(4\,\text{cis}(75°)\right)^2$
    c. $\left(2\,\text{cis}(\pi/18)\right)^6$

35. Find the indicated roots of the complex number. Write your answers in standard form.
    a. Square roots of $9i$
    b. Cube roots of $-8i$
    c. Fourth roots of $-8+8\sqrt{3}i$