

1a. Answers may vary. Typical response:

$$\frac{7\pi}{9} + \frac{18\pi}{9} = \frac{25\pi}{9} \quad \text{and} \quad \frac{7\pi}{9} - \frac{18\pi}{9} = \frac{11\pi}{9}$$

1b. Answers may vary. Typical response:

$$-\frac{3\pi}{7} + \frac{14\pi}{7} = \frac{11\pi}{7} \quad \text{and} \quad -\frac{3\pi}{7} - \frac{14\pi}{7} = \frac{17\pi}{7}$$

2a.
$$\frac{235^\circ}{180^\circ} \cdot \frac{\pi \text{ rad}}{1} = \frac{47}{36} \cdot \frac{\pi \text{ rad}}{1} = \frac{47\pi}{36} \text{ rad}$$

2b.
$$\frac{-100^\circ}{180^\circ} \cdot \frac{\pi \text{ rad}}{1} = \frac{-5}{9} \cdot \frac{\pi \text{ rad}}{1} = \frac{5\pi}{9} \text{ rad}$$

3a.
$$\frac{4\pi \text{ rad}}{15} \cdot \frac{180^\circ}{\pi \text{ rad}} = \frac{4}{15} \cdot 180^\circ = 48^\circ$$

3b.
$$\frac{-7\pi \text{ rad}}{6} \cdot \frac{180^\circ}{\pi \text{ rad}} = \frac{-7}{6} \cdot 180^\circ = -210^\circ$$

4a.
$$\theta = \frac{s}{r} = \frac{15 \text{ cm}}{12 \text{ cm}} = \frac{5}{4} \text{ rad}$$

4b.
$$\theta = \frac{s}{r} = \frac{10 \text{ ft}}{24 \text{ ft}} = \frac{5}{12} \text{ rad}$$

5a.
$$s = r\theta = (5 \text{ in}) \left(\frac{3\pi}{10} \text{ rad} \right) = \frac{3\pi}{2} \text{ in}$$

5b.
$$s = r\theta = (60 \text{ m}) \left(\frac{7\pi}{24} \text{ rad} \right) = \frac{35\pi}{2} \text{ m}$$

6a.
$$r = \frac{s}{\theta} = \frac{6 \text{ yd}}{\frac{2\pi}{3} \text{ rad}} \cdot \frac{3}{3} = \frac{18 \text{ yd}}{2\pi \text{ rad}} = \frac{9}{\pi} \text{ yd}$$

6b.
$$r = \frac{s}{\theta} = \frac{14 \text{ mm}}{\frac{5\pi}{7} \text{ rad}} \cdot \frac{7}{7} = \frac{98 \text{ mm}}{5\pi \text{ rad}}$$

7a.
$$\frac{5000 \text{ rev}}{1 \text{ min}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = \frac{500}{1} \cdot \frac{\pi \text{ rad}}{1} \cdot \frac{1}{3 \text{ sec}} = \frac{500\pi}{3} \text{ rad/sec}$$

7b.
$$\frac{5000 \text{ rev}}{1 \text{ min}} \cdot \frac{8\pi \text{ in}}{1 \text{ rev}} \cdot \frac{1 \text{ ft}}{12 \text{ in}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = \frac{250}{1} \cdot \frac{2\pi}{1} \cdot \frac{1 \text{ ft}}{3} \cdot \frac{1}{3 \text{ sec}} = \frac{500\pi}{9} \text{ ft/sec}$$

8a.
$$\frac{1600 \text{ rev}}{1 \text{ min}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = \frac{160}{1} \cdot \frac{\pi \text{ rad}}{1} \cdot \frac{1}{3 \text{ sec}} = \frac{160\pi}{3} \text{ rad/sec}$$

8b.
$$\frac{1600 \text{ rev}}{1 \text{ min}} \cdot \frac{3\pi \text{ ft}}{1 \text{ rev}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = \frac{80}{1} \cdot \frac{\pi \text{ ft}}{1} \cdot \frac{1}{1 \text{ sec}} = 80\pi \text{ ft/sec}$$

$$9a. \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

$$\csc\left(\frac{\pi}{6}\right) = 2$$

$$9b. \sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$\csc\left(\frac{2\pi}{3}\right) = \frac{2}{\sqrt{3}} \text{ or } \frac{2\sqrt{3}}{3}$$

$$9c. \sin\left(\frac{5\pi}{4}\right) = -\frac{1}{\sqrt{2}} \text{ or } -\frac{\sqrt{2}}{2}$$

$$\csc\left(\frac{5\pi}{4}\right) = -\sqrt{2} \text{ or } -\frac{2}{\sqrt{2}}$$

$$9d. \sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$$

$$\csc\left(-\frac{\pi}{6}\right) = -2$$

$$9e. \sin(\pi) = 0$$

$$\csc(\pi) \text{ is } \text{undef}$$

$$9f. \sin\left(\frac{3\pi}{2}\right) = -1$$

$$\csc\left(\frac{3\pi}{2}\right) = -1$$

$$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

$$\sec\left(\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}} \text{ or } \frac{2\sqrt{3}}{3}$$

$$\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$$

$$\sec\left(\frac{2\pi}{3}\right) = -2$$

$$\cos\left(\frac{5\pi}{4}\right) = -\frac{1}{\sqrt{2}} \text{ or } -\frac{\sqrt{2}}{2}$$

$$\sec\left(\frac{5\pi}{4}\right) = -\sqrt{2} \text{ or } -\frac{2}{\sqrt{2}}$$

$$\cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

$$\sec\left(-\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}} \text{ or } \frac{2\sqrt{3}}{3}$$

$$\cos(\pi) = -1$$

$$\sec(\pi) = -1$$

$$\cos\left(\frac{3\pi}{2}\right) = 0$$

$$\sec\left(\frac{3\pi}{2}\right) \text{ is } \text{undef}$$

$$\tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}} \text{ or } \frac{\sqrt{3}}{3}$$

$$\cot\left(\frac{\pi}{6}\right) = \sqrt{3} \text{ or } \frac{3}{\sqrt{3}}$$

$$\tan\left(\frac{2\pi}{3}\right) = -\sqrt{3}$$

$$\cot\left(\frac{2\pi}{3}\right) = -\frac{1}{\sqrt{3}} \text{ or } -\frac{\sqrt{3}}{3}$$

$$\tan\left(\frac{5\pi}{4}\right) = 1$$

$$\cot\left(\frac{5\pi}{4}\right) = 1$$

$$\tan\left(-\frac{\pi}{6}\right) = -\frac{1}{\sqrt{3}} \text{ or } -\frac{\sqrt{3}}{3}$$

$$\cot\left(-\frac{\pi}{6}\right) = -\sqrt{3} \text{ or } -\frac{3}{\sqrt{3}}$$

$$\tan(\pi) = 0$$

$$\cot(\pi) \text{ is } \text{undef}$$

$$\tan\left(\frac{3\pi}{2}\right) \text{ is } \text{undef}$$

$$\cot\left(\frac{3\pi}{2}\right) = 0$$

$$10. \text{ opp} = 8, \text{ adj} = 15, \text{ hyp} = 17$$

$$\sin(\theta) = \frac{8}{17}$$

$$\csc(\theta) = \frac{17}{8}$$

$$\cos(\theta) = \frac{15}{17}$$

$$\sec(\theta) = \frac{17}{15}$$

$$\tan(\theta) = \frac{8}{15}$$

$$\cot(\theta) = \frac{15}{8}$$

$$11a. \text{ opp} = 5, \text{ adj} = 12, \text{ hyp} = 13$$

$$\csc(\theta) = \frac{13}{5}$$

$$\cos(\theta) = \frac{12}{13}$$

$$\sec(\theta) = \frac{13}{12}$$

$$\tan(\theta) = \frac{5}{12}$$

$$\cot(\theta) = \frac{12}{5}$$

$$11b. \text{ opp} = \sqrt{3}, \text{ adj} = 5, \text{ hyp} = 2\sqrt{7}$$

$$\sin(\theta) = \frac{\sqrt{3}}{2\sqrt{7}} \text{ or } \frac{\sqrt{21}}{14}$$

$$\csc(\theta) = \frac{2\sqrt{7}}{\sqrt{3}} \text{ or } \frac{2\sqrt{21}}{3} \text{ or } \frac{14}{\sqrt{21}}$$

$$\cos(\theta) = \frac{5}{2\sqrt{7}} \text{ or } \frac{5\sqrt{7}}{14}$$

$$\sec(\theta) = \frac{2\sqrt{7}}{5} \text{ or } \frac{14}{5\sqrt{7}}$$

$$\cot(\theta) = \frac{5}{\sqrt{3}} \text{ or } \frac{5\sqrt{3}}{3}$$

12a. $\csc(\theta) = \frac{3}{2}$

$$\begin{aligned}\sin^2(\theta) + \cos^2(\theta) &= 1 \\ \cos^2(\theta) &= 1 - \sin^2(\theta)\end{aligned}$$

$$\cos^2(\theta) = 1 - \left(\frac{2}{3}\right)^2$$

$$\cos^2(\theta) = 1 - \frac{4}{9}$$

$$\cos^2(\theta) = \frac{5}{9}$$

$$\cos(\theta) = \frac{\sqrt{5}}{3}$$

$$\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

$$= \frac{\frac{\sqrt{5}}{3}}{\frac{2}{3}} \cdot \frac{3}{3}$$

$$= \frac{\sqrt{5}}{2}$$

12b. $\cos(\theta) = \frac{4}{5}$

$$\begin{aligned}\sin^2(\theta) + \cos^2(\theta) &= 1 \\ \sin^2(\theta) &= 1 - \cos^2(\theta)\end{aligned}$$

$$\sin^2(\theta) = 1 - \left(\frac{4}{5}\right)^2$$

$$\sin^2(\theta) = 1 - \frac{16}{25}$$

$$\sin^2(\theta) = \frac{9}{25}$$

$$\sin(\theta) = \frac{3}{5}$$

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{\frac{3}{5}}{\frac{4}{5}} \cdot \frac{5}{5} = \frac{3}{4}$$

13. $x = -8, y = -15, r = 17$

$$\sin(\theta) = \frac{-15}{17}$$

$$\csc(\theta) = \frac{-17}{15}$$

$$\cos(\theta) = \frac{-8}{17}$$

$$\sec(\theta) = \frac{-17}{8}$$

$$\tan(\theta) = \frac{15}{8}$$

$$\cot(\theta) = \frac{8}{15}$$

14a. $x = -12, y = 5, r = 13$

$$\csc(\theta) = \frac{13}{5}$$

$$\cos(\theta) = \frac{-12}{13}$$

$$\sec(\theta) = \frac{-13}{12}$$

$$\tan(\theta) = \frac{-5}{12}$$

$$\cot(\theta) = \frac{-12}{5}$$

14b. $x = 5, y = -4, r = \sqrt{41}$

$$\sin(\theta) = \frac{4}{\sqrt{41}} \text{ or } \frac{4\sqrt{41}}{41}$$

$$\csc(\theta) = \frac{\sqrt{41}}{4} \text{ or } \frac{41}{4\sqrt{41}}$$

$$\cos(\theta) = \frac{5}{\sqrt{41}} \text{ or } \frac{5\sqrt{41}}{41}$$

$$\sec(\theta) = \frac{\sqrt{41}}{5} \text{ or } \frac{41}{5\sqrt{41}}$$

$$\tan(\theta) = \frac{4}{5}$$

$$15a. \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \text{ or } \frac{\sqrt{2}}{2}$$

$$\csc\left(\frac{\pi}{4}\right) = \sqrt{2} \text{ or } \frac{2}{\sqrt{2}}$$

$$15b. \sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$$

$$\csc\left(\frac{5\pi}{6}\right) = 2$$

$$15c. \sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

$$\csc\left(\frac{4\pi}{3}\right) = -\frac{2}{\sqrt{3}} \text{ or } -\frac{2\sqrt{3}}{3}$$

$$15d. \sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

$$\csc\left(-\frac{\pi}{3}\right) = -\frac{2}{\sqrt{3}} \text{ or } -\frac{2\sqrt{3}}{3}$$

$$15e. \sin(0) = 0$$

$$\csc(0) \text{ is } \text{undef}$$

$$15f. \sin\left(\frac{9\pi}{2}\right) = 1$$

$$\csc\left(\frac{9\pi}{2}\right) = 1$$

$$\cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \text{ or } \frac{\sqrt{2}}{2}$$

$$\sec\left(\frac{\pi}{4}\right) = \sqrt{2} \text{ or } \frac{2}{\sqrt{2}}$$

$$\cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$$

$$\sec\left(\frac{5\pi}{6}\right) = -\frac{2}{\sqrt{3}} \text{ or } -\frac{2\sqrt{3}}{3}$$

$$\cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2}$$

$$\sec\left(\frac{4\pi}{3}\right) = -2$$

$$\cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}$$

$$\sec\left(-\frac{\pi}{3}\right) = 2$$

$$\cos(0) = 1$$

$$\sec(0) = 1$$

$$\cos\left(\frac{9\pi}{2}\right) = 0$$

$$\sec\left(\frac{9\pi}{2}\right) \text{ is } \text{undef}$$

$$\tan\left(\frac{\pi}{4}\right) = 1$$

$$\cot\left(\frac{\pi}{4}\right) = 1$$

$$\tan\left(\frac{5\pi}{6}\right) = -\frac{1}{\sqrt{3}} \text{ or } -\frac{\sqrt{3}}{3}$$

$$\cot\left(\frac{5\pi}{6}\right) = -\sqrt{3} \text{ or } -\frac{3}{\sqrt{3}}$$

$$\tan\left(\frac{4\pi}{3}\right) = \sqrt{3}$$

$$\cot\left(\frac{4\pi}{3}\right) = \frac{1}{\sqrt{3}} \text{ or } \frac{\sqrt{3}}{3}$$

$$\tan\left(-\frac{\pi}{3}\right) = -\sqrt{3}$$

$$\cot\left(-\frac{\pi}{3}\right) = -\frac{1}{\sqrt{3}} \text{ or } -\frac{\sqrt{3}}{3}$$

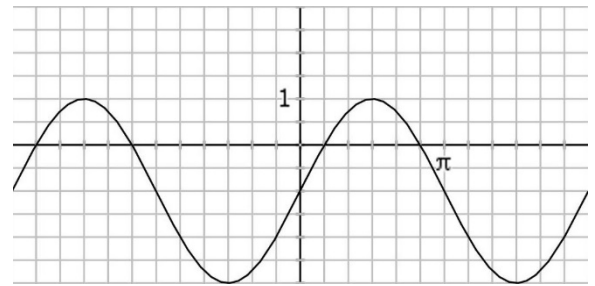
$$\tan(0) = 0$$

$$\cot(0) \text{ is } \text{undef}$$

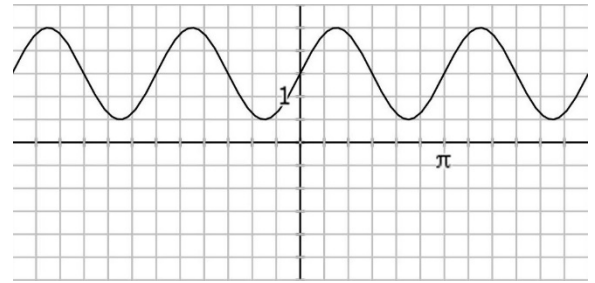
$$\tan\left(\frac{9\pi}{2}\right) \text{ is } \text{undef}$$

$$\cot\left(\frac{9\pi}{2}\right) = 0$$

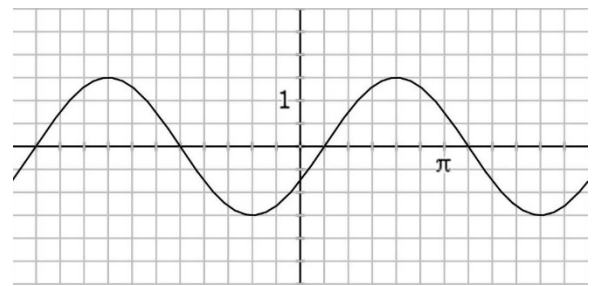
16a.	amplitude 2	period 2π	PS none	VS 1 down
	$x = 0$ sq	$x = 1$ sq	$x = 3$ sq	
	$y = 2(0) - 1$	$y = 2(0.5) - 1$	$y = 2(1) - 1$	
	$= 0 - 1$	$= 1 - 1$	$= 2 - 1$	
	$y = -1$	$y = 0$	$y = 1$	



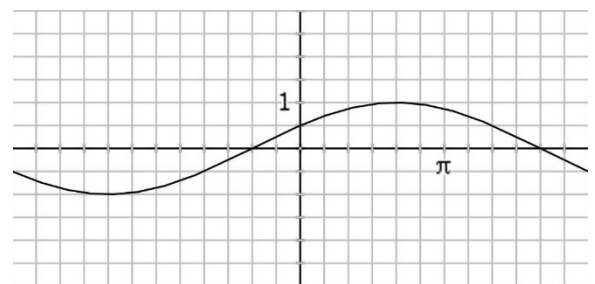
16b.	amplitude 1	period π	PS none	VS 1.5 up
	$2x = 0$	$2x = 1$	$2x = 3$	
	$x = 0$ sq	$x = 0.5$ sq	$x = 1.5$ sq	
	$y = 0 + 1.5$	$y = 0.5 + 1.5$	$y = 1 + 1.5$	
	$y = 1.5$	$y = 2$	$y = 2.5$	



16c.	amplitude 1.5	period 2π	PS $\pi/3$ left	VS none
	$x + 2 = 3$	$x + 2 = 2$	$x + 2 = 0$	
	$x = 1$ sq	$x = 0$ sq	$x = -2$ sq	
	$y = -1.5(0)$	$y = -1.5(0.5)$	$y = -1.5(1)$	
	$y = 0$	$y = -0.75$	$y = -1.5$	

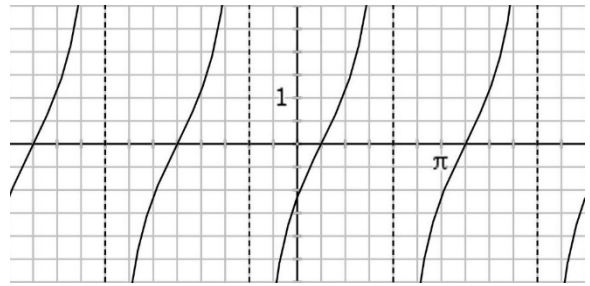


16d.	amplitude 1	period 4π	PS $2\pi/3$ right	VS none
	$0.5x - 2 = 3$	$0.5x - 2 = 2$	$0.5x - 2 = 0$	
	$0.5x = 5$	$0.5x = 4$	$0.5x = 2$	
	$x = 10$ sq	$x = 8$ sq	$x = 4$ sq	
	$y = 0$	$y = 0.5$	$y = 1$	



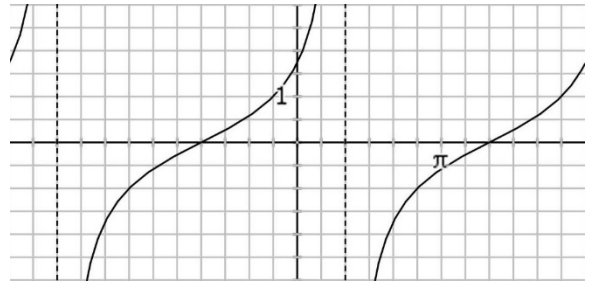
17a. amplitude 2 period π PS $\pi/6$ right

$x-1=3$	$x-1=0$	$x-1=1.5$
$x=4$ sq	$x=1$ sq	$x=2.5$ sq
	$y=2(0)$	$y=2(1)$
	$y=0$	$y=2$



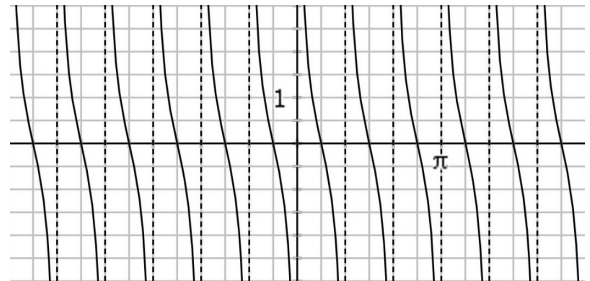
17b. amplitude 1 period 2π PS $2\pi/3$ left

$0.5x+2=3$	$0.5x+2=0$	$0.5x+2=1.5$
$0.5x=1$	$0.5x=-2$	$0.5x=-0.5$
$x=2$ sq	$x=-4$ sq	$x=-1$ sq
	$y=0$	$y=1$



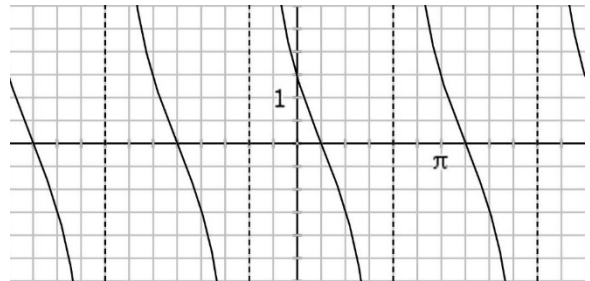
17c. amplitude 1.5 period $\pi/3$ PS none

$3x=0$	$3x=3$	$3x=1.5$
$x=0$	$x=1$ sq	$x=0.5$ sq
	$y=1.5(0)$	$y=1.5(1)$
	$y=0$	$y=1.5$



17d. amplitude 2.5 period π PS $\pi/3$ left

$x+2=0$	$x+2=3$	$x+2=1.5$
$x=-2$ sq	$x=1$ sq	$x=-0.5$ sq
	$y=2.5(0)$	$y=2.5(1)$
	$y=0$	$y=2.5$

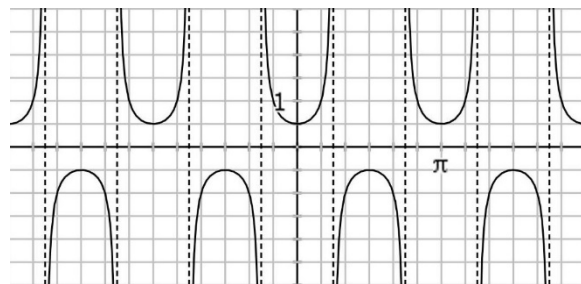


18a.

period π	PS none
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$2x = 3$	$2x = 0$
$x = 1.5$ sq	$x = 0$ sq
	$y = 0.5(1)$
	$y = 0.5$

$2x = 2$
 $x = 1$ sq
 $y = 0.5(2)$
 $y = 1$

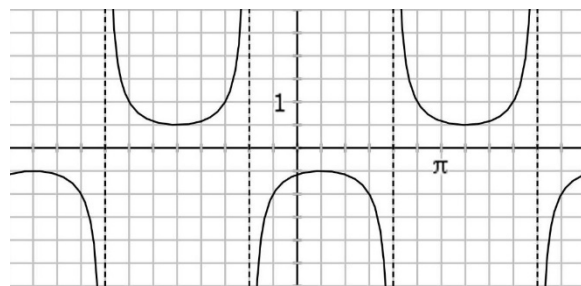


18b.

period 2π	PS $\pi/6$ right
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$x - 1 = 3$	$x - 1 = 0$
$x = 4$ sq	$x = 1$ sq
	$y = -0.5(1)$
	$y = -0.5$

$x - 1 = 2$
 $x = 3$ sq
 $y = -0.5(2)$
 $y = -1$

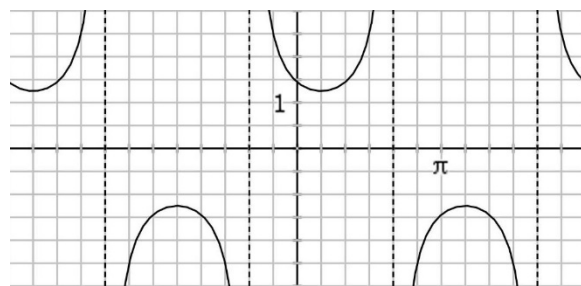


18c.

period 2π	PS $\pi/3$ left
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$x + 2 = 0$	$x + 2 = 3$
$x = -2$ sq	$x = 1$ sq
	$y = 1(1.25)$
	$y = 1.25$

$x + 2 = 1$
 $x = -1$ sq
 $y = 1.25(2)$
 $y = 2.5$

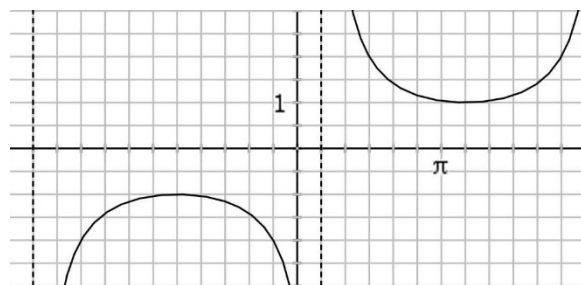


18d.

period 4π	PS $\pi/6$ right
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$0.5x - 0.5 = 0$	$0.5x - 0.5 = 3$
$0.5x = 0.5$	$0.5x = 3.5$
$x = 1$ sq	$x = 7$ sq
	$y = 1$

$0.5x - 0.5 = 1$
 $0.5x = 1.5$
 $x = 3$ sq
 $y = 2$



$$19a. \arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

$$19d. \cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$$

$$19g. \arctan(\sqrt{3}) = \frac{\pi}{3}$$

$$20a. \sin(\arcsin(-0.7)) = -0.7$$

$$20d. \arccos\left(\cos\left(\frac{5\pi}{3}\right)\right) = \frac{\pi}{3}$$

$$21a. x = 12, y = -5, r = 13$$

$$\cos\left(\arctan\left(-\frac{5}{12}\right)\right) = \frac{12}{13}$$

$$\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$$

$$\arccos\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$$

$$\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{6}$$

$$\sin^{-1}\left(\sin\left(-\frac{\pi}{6}\right)\right) = -\frac{\pi}{6}$$

$$\tan^{-1}\left(\tan\left(\frac{4\pi}{3}\right)\right) = \frac{\pi}{3}$$

$$x = \sqrt{7}, y = 3, r = 4$$

$$\sec\left(\arcsin\left(\frac{3}{4}\right)\right) = \frac{4}{\sqrt{7}} \text{ or } \frac{4\sqrt{7}}{7}$$

$$\arcsin(-1) = -\frac{\pi}{2}$$

$$\cos^{-1}(-1) = \pi$$

$$\arctan(-1) = -\frac{\pi}{4}$$

$$\cos^{-1}\left(\cos\left(\frac{\pi}{4}\right)\right) = \frac{\pi}{4}$$

$$\arctan\left(\tan\left(\frac{11\pi}{6}\right)\right) = -\frac{\pi}{6}$$

$$x = -\sqrt{5}, y = 2, r = 3$$

$$\csc\left(\arccos\left(-\frac{\sqrt{5}}{3}\right)\right) = \frac{3}{2}$$