Trigonometry/Precalculus

§9.3b Hyperbolas

Name ______________________

Date ______________ Pd ______

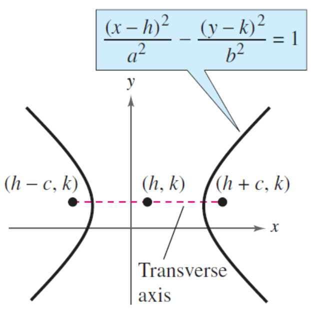
Standard Equation of a Hyperbola

The **standard form of the equation of a hyperbola** with center at (h, k) is

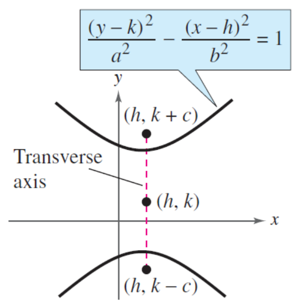
$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1 \qquad \text{(horizontal transverse axis)}$$

$$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1. \qquad \text{(vertical transverse axis)}$$

The vertices are a units from the center, and the foci are c units from the center. Moreover, $c^2=a^2+b^2$.



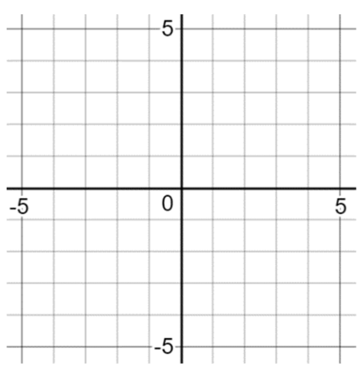
Transverse axis is horizontal.



Transverse axis is vertical.

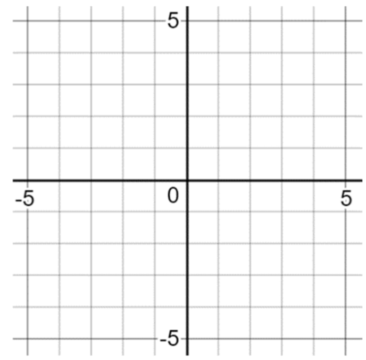
Example 1

Rewrite the equation of the hyperbola $4x^2-y^2=16$ in standard form. Sketch its graph, and identify the center, vertices, foci, and asymptotes.



Example 2

Rewrite the equation of the hyperbola $4x^2-y^2+8x+8=0$ in standard form. Sketch its graph, and identify the center, vertices, foci, and asymptotes.



Applications

The following application was developed during World War II. It shows how the properties of hyperbolas can be used in radar and other detection systems.

Example 3

Two microphones, 1 mile (5280 feet) apart, record an explosion. Microphone A receives the sound 2 seconds before microphone B. Where did the explosion occur? Assume sound travels at 1100 feet per second.

Comet Orbits

Another interesting application of conic sections involves the orbits of comets in our solar system. Of the 610 comets identified prior to 1970, 245 have elliptical orbits, 295 have parabolic orbits, and 70 have hyperbolic orbits. The center of the sun is a focus of each of these orbits, and each orbit has a vertex at the point where the comet is closest to the sun.

Undoubtedly, there are many comets with parabolic or hyperbolic orbits that have not been identified. You get to see such comets only *once*. Comets with elliptical orbits, such as Halley's comet, are the only ones that remain in our solar system.

If p is the distance between the vertex and the focus in meters, and v is the velocity of the comet at the vertex in meters per second, then the type of orbit is determined as follows.

1. Ellipse: $v < \sqrt{2GM/p}$
2. Parabola: $v = \sqrt{2GM/p}$
3. Hyperbola: $v > \sqrt{2GM/p}$

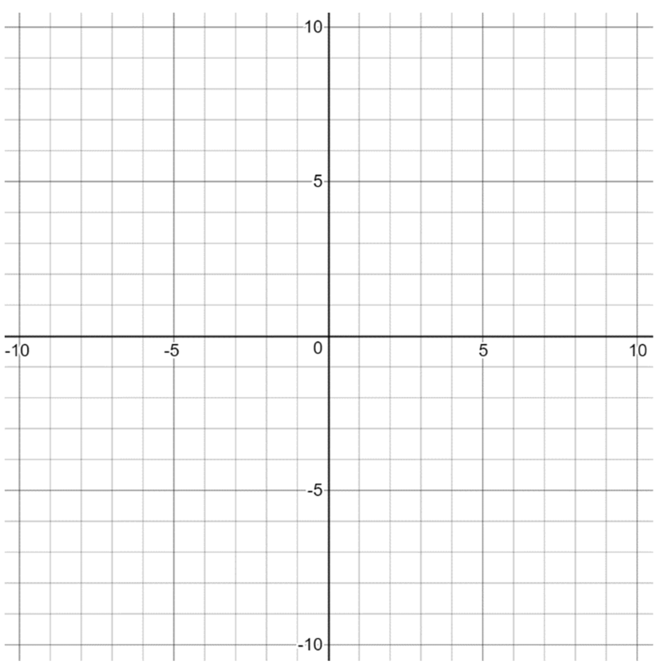
In each of these equations, $M \approx 1.989 \times 10^{30}$ kilograms (the mass of the sun) and $G \approx 6.67 \times 10^{-11}$ cubic meter per kilogram-second squared (the universal gravitational constant).

Trigonometry/Precalculus | Name ____________________

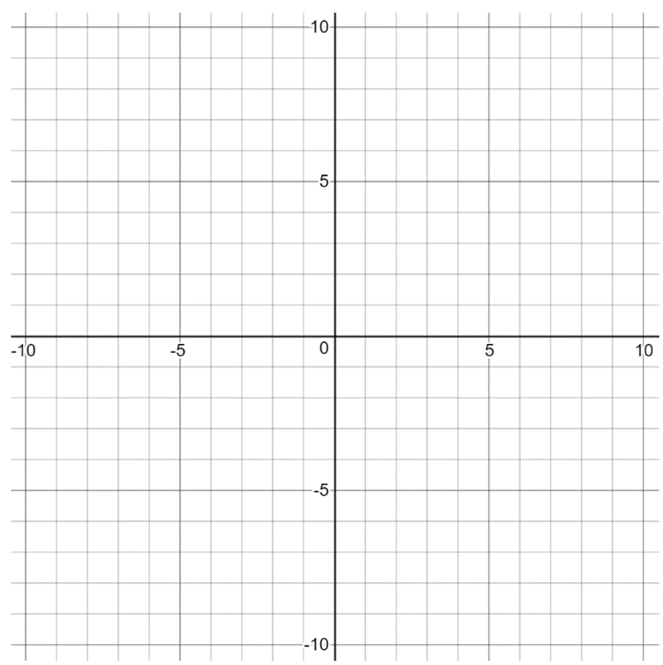
§9.3b Hyperbolas HOMEWORK | Date ____________ Pd ______

In Exercises 1-3, rewrite the equation of the hyperbola in standard form. Sketch its graph, and identify the center, vertices, foci, and asymptotes.

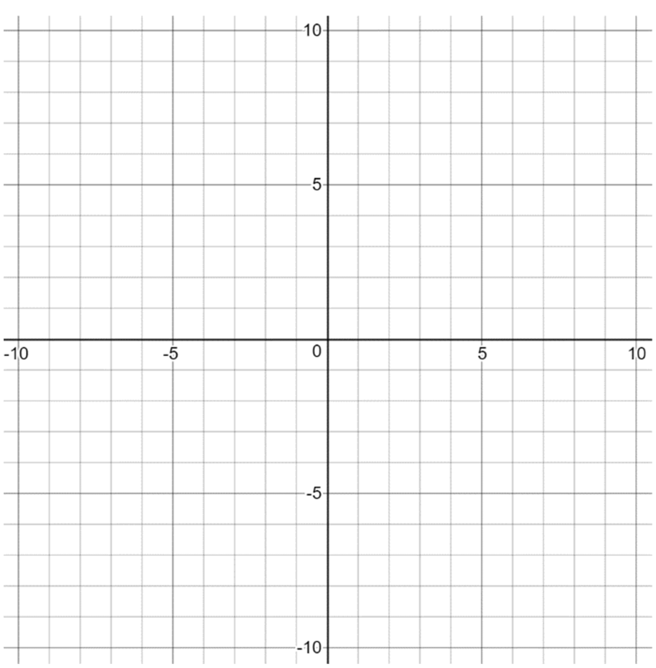
1. $9x^2 - y^2 - 36x - 6y + 18 = 0$



2. $x^2 - 9y^2 + 2x - 54y - 81 = 0$



3. $16y^2 - x^2 + 2x + 64y - 1 = 0$



4. A hyperbolic mirror (used in some telescopes) has the property that a light ray directed at a focus will be reflected to the other focus. The focus of a hyperbolic mirror (see figure) has coordinates (24, 0). Find the vertex of the mirror if the mount at the top edge of the mirror has coordinates (24, 24).