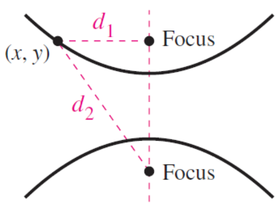
Trigonometry/Precalculus
§9.3a Hyperbolas

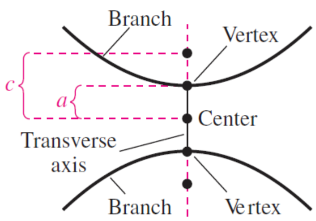
Name ______________________
Date ______________ Pd ______

Definition of a Hyperbola

A **hyperbola** is the set of all points (x, y) in a plane, the difference of whose distances from two distinct fixed points, the **foci**, is a positive constant.



$d_2 - d_1$ is a positive constant.



The graph of a hyperbola has two disconnected parts called the **branches**. The line through the two foci intersect the hyperbola at two points called the **vertices**. The line segment connecting the vertices is the **transverse axis**, and the midpoint of the transverse axis is the **center** of the hyperbola.

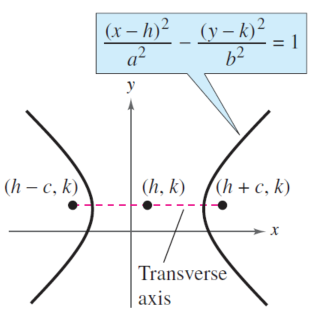
Standard Equation of a Hyperbola

The **standard form of the equation of a hyperbola** with center at (h, k) is

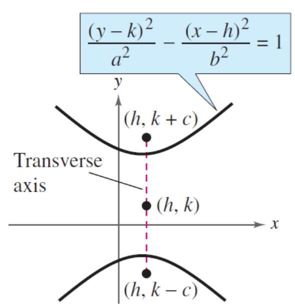
$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \qquad \text{(horizontal transverse axis)}$$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1. \qquad \text{(vertical transverse axis)}$$

The vertices are a units from the center, and the foci are c units from the center. Moreover, $c^2 = a^2 + b^2$.



Transverse axis is horizontal.



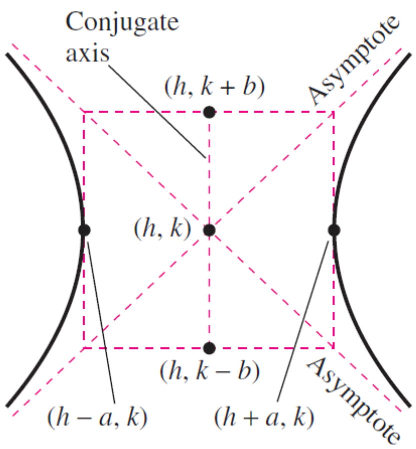
Transverse axis is vertical.

Example 1

Find the standard form of the equation of the hyperbola with foci $(-1, 2)$ and $(5, 2)$, and vertices $(0, 2)$ and $(4, 2)$. Identify its center.

Asymptotes of a Hyperbola

Each hyperbola has two **asymptotes** that intersect at the center of the hyperbola. The asymptotes pass through the corners of a rectangle of dimensions $2a$ and $2b$, with its center at (h, k).



Asymptotes of a Hyperbola

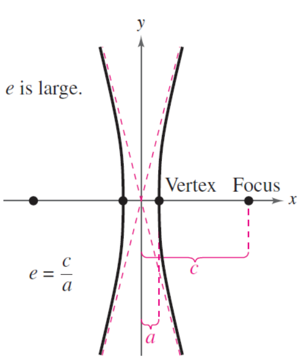
$$y = k \pm \frac{b}{a}(x - h) \qquad \text{(horizontal transverse axis)}$$

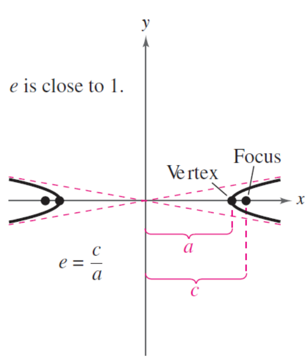
$$y = k \pm \frac{a}{b}(x - h) \qquad \text{(vertical transverse axis)}$$

The **conjugate axis** of a hyperbola is the line segment of length $2b$ joining $(h, k+b)$ and $(h, k-b)$ if the transverse axis is horizontal, and the line segment of length $2b$ joining $(h+b, k)$ and $(h-b, k)$ if the transverse axis is vertical.

Eccentricity

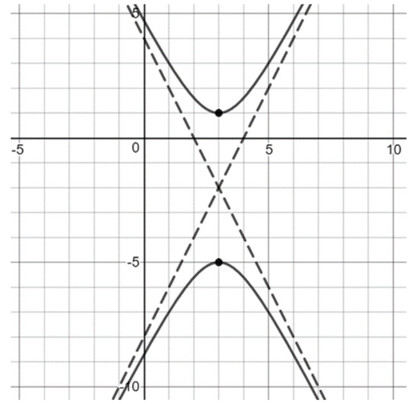
As with ellipses, the *eccentricity* of a hyperbola is $e = \frac{c}{a}$, and because $c > a$ it follows that $e > 1$. If the eccentricity is large, the branches of the hyperbola are nearly flat. If the eccentricity is close to 1, the branches of the hyperbola are more pointed.





Example 2

Find the standard form of the equation of the hyperbola having vertices $(3, -5)$ and $(3, 1)$ and having asymptotes $y = 2x - 8$ and $y = -2x + 4$.



Trigonometry/Precalculus

§9.3a Hyperbolas HOMEWORK

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In Exercises 1-8, find the standard form of the equation of the hyperbola with the given characteristics. Identify its center, vertices, foci, and asymptotes.

1. vertices $(0, \pm 2)$; foci $(0, \pm 4)$

2. foci $(\pm 10, 0)$; asymptotes $y = \pm \frac{3}{4}x$

3. vertices $(2, 0)$, $(6, 0)$; foci $(0, 0)$, $(8, 0)$

4. vertices $(4, 1)$, $(4, 9)$; foci $(4, 0)$, $(4, 10)$

5. vertices $(1, 2)$, $(3, 2)$; asymptotes $y = x$, $y = 4 - x$

6. vertices $(3, 0)$, $(3, 4)$; asymptotes $y = \frac{2}{3}x$, $y = 4 - \frac{2}{3}x$

7. vertices $(-2, 1)$, $(2, 1)$; passes through the point $(5, 4)$

8. vertices $(0, 4)$, $(0, 0)$; passes through the point $(\sqrt{5}, -1)$