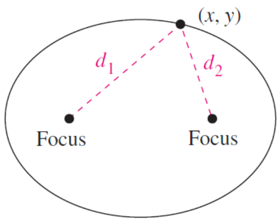
Trigonometry/Precalculus — Name ______________________

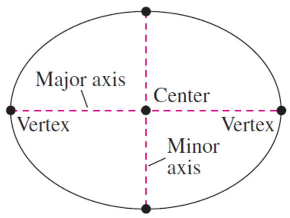
§9.2a Ellipses — Date ______________ Pd ______

Definition of an Ellipse

An **ellipse** is the set of all points (x, y) in a plane, the sum of whose distances from two distinct fixed points (**foci**) is constant.

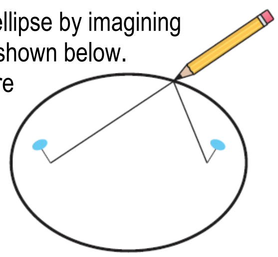


$d_1 + d_2$ is constant.



The line through the foci intersect the ellipse at two points called **vertices**. The chord joining the vertices is the **major axis**, and its midpoint is the **center** of the ellipse. The chord perpendicular to the major axis at the center is the **minor axis**.

You can visualize the definition of an ellipse by imagining two thumbtacks placed at the foci, as shown below. If the ends of a fixed length of string are fastened to the thumbtacks and the string is drawn taut with a pencil, the path traced by the pencil will be an ellipse.



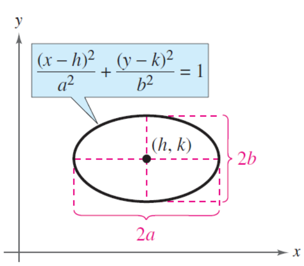
Standard Equation of an Ellipse

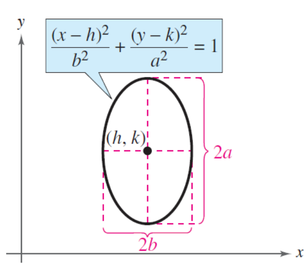
The **standard form of the equation of an ellipse** with center (h, k) and major and minor axes of lengths $2a$ and $2b$, respectively, where $0 < b < a$, is

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \qquad \text{(major axis is horizontal)}$$

$$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1. \qquad \text{(major axis is vertical)}$$

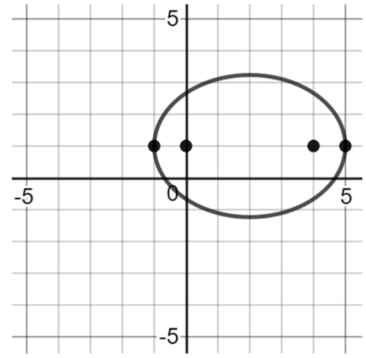
The foci lie on the major axis, c units from the center, with $c^2 = a^2 - b^2$.





Example 1

Write the standard form of the equation of the ellipse shown. Identify its center, vertices, and foci.

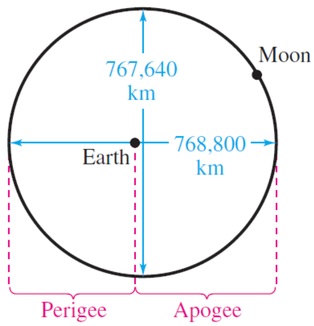


Application

Ellipses have many practical and aesthetic uses. For instance, machine gears, supporting arches, and acoustic designs often involve elliptical shapes. The orbits of satellites and planets are also ellipses.

Example 2

The moon travels about Earth in an elliptical orbit with Earth at one focus, as shown in the figure. The major and minor axes of the orbit have lengths of 768,800 kilometers and 767,640 kilometers, respectively. Find the greatest and smallest distances (the *apogee* and *perigee*) from Earth's center to the moon's center



Eccentricity

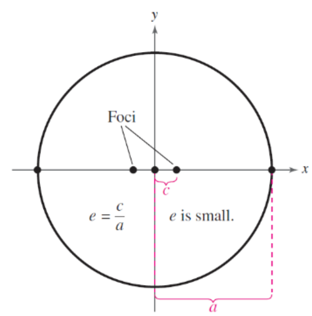
One of the reasons it was difficult for early astronomers to detect that the orbits of the planets are ellipses is that the foci of the planetary orbits are relatively close to their centers, and so the orbits are nearly circular. To measure the ovalness of an ellipse, you can use the concept of eccentricity.

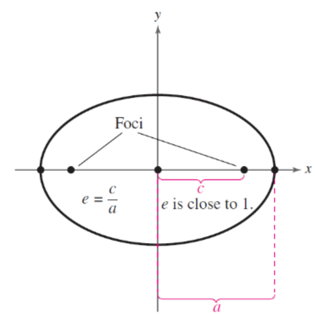
Definition of Eccentricity

The **eccentricity** e of an ellipse is given by the ratio $e = \frac{c}{a}$.

Note that $0 < e < 1$ for *every* ellipse.

For an ellipse that is nearly circular, the foci are close to the center and the ratio c/a is small. On the other hand, for an elongated ellipse, the foci are close to the vertices and the ratio c/a is close to 1.





The orbit of the moon has an eccentricity of $e \approx 0.0549$, and the eccentricities of the eight planetary orbits are as follows.

Mercury:	$e \approx 0.2056$
Venus:	$e \approx 0.0068$
Earth:	$e \approx 0.0167$
Mars:	$e \approx 0.0934$
Jupiter:	$e \approx 0.0484$
Saturn:	$e \approx 0.0542$
Uranus:	$e \approx 0.0472$
Neptune:	$e \approx 0.0086$

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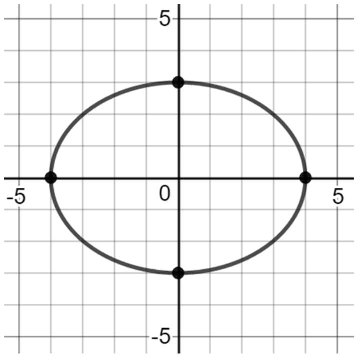
In Exercises 1-4, write the standard form of the equation of the ellipse shown. Identify its center, vertices, foci, and eccentricity

1.



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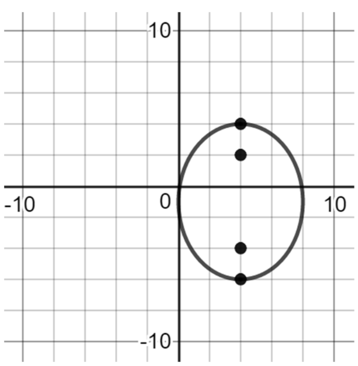

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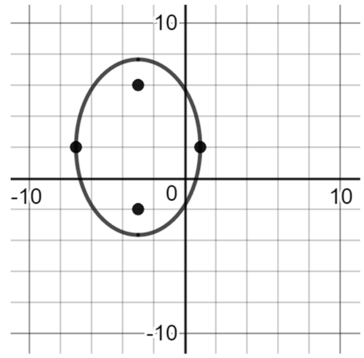
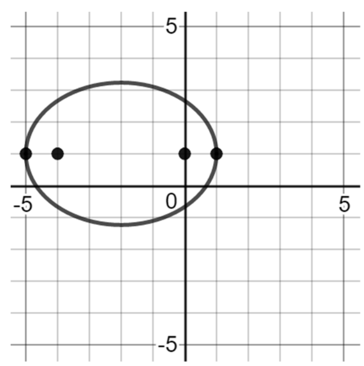


4.



5. A semielliptical arch over a tunnel for a road through a mountain has a major axis of 100 feet and a height at the center of 40 feet.

   Suppose the center of the road entering the tunnel is at the origin. Find an equation of the semielliptical arch over the tunnel.

6. Statuary Hall is an elliptical room in the United States Capitol Building in Washington, D.C. The room is also referred to as the Whispering Gallery because a person standing at one focus of the room can hear even a whisper spoken by a person standing at the other focus.

   Given that the dimensions of Statuary Hall are 46 feet wide by 97 feet long, find an equation for the shape of the floor surface of the hall.