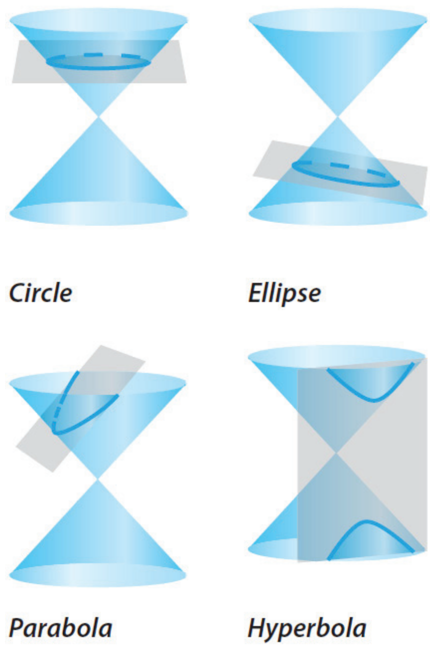
Trigonometry/Precalculus | Name ____________________

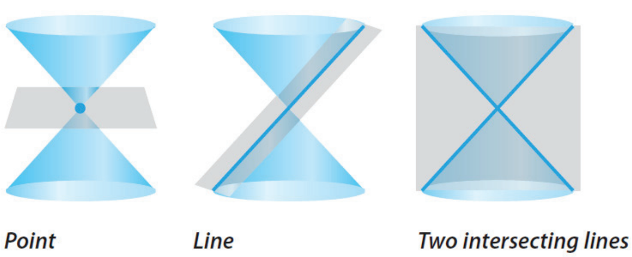
§9.1 Circles and Parabolas | Date ______________ Pd ______

Conics

A **conic section** (or simply **conic**) is the intersection of a plane and a double-napped cone. Notice in the figure that in the formation of the four basic conics, the intersecting plane does not pass through the vertex of the cone.



When the plane does pass through the vertex, the resulting figure is a **degenerate conic**, as shown in the figure below.



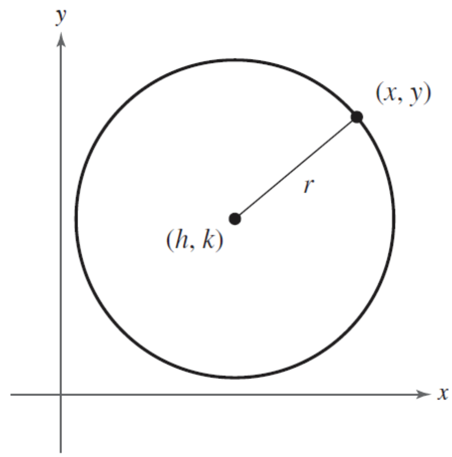
There are several ways to approach the study of conics. You could begin by defining conics in terms of the intersection of planes and cones, as the Greeks did (600 to 300 B.C.), or you could define them algebraically, in terms of the general second-degree equation

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0.$$

However, you will study a third approach, in which each of the conic is defined as a **locus** (collection) of points satisfying a certain geometric property.

Definition of a Circle

A **circle** is the set of all points (x, y) in a plane that are equidistant from a fixed point (h, k), called the **center** of the circle. The distance r between the center and any point (x, y) on the circle is the **radius**.



Standard Form of the Equation of a Circle

The **standard form of the equation of a circle** is

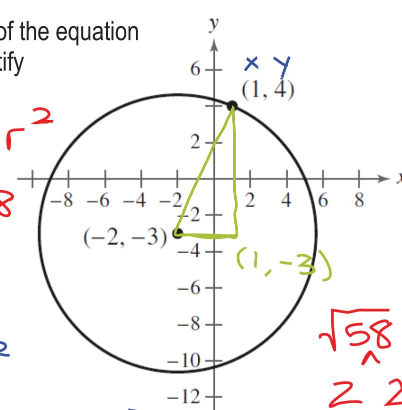
$$(x-h)^2 + (y-k)^2 = r^2.$$

The point (h, k) is the center of the circle, and the positive number r is the radius of the circle. The standard form of the equation of a circle whose center is the origin, $(h, k) = (0, 0)$, is

$$x^2 + y^2 = r^2.$$

Example 1

Write the standard form of the equation of the circle shown. Identify its center and radius.



x y

$(x-h)^2+(y-k)^2=r^2$

$(x+2)^2+(y+3)^2=58$

center: $(-2,-3)$
h k

(1, -3)

$(1+2)^2+(4+3)^2=r^2$

$3^2+7^2=r^2$

$9+49=r^2$

$58=r^2$

radius $\sqrt{58}$

$\sqrt{58}$

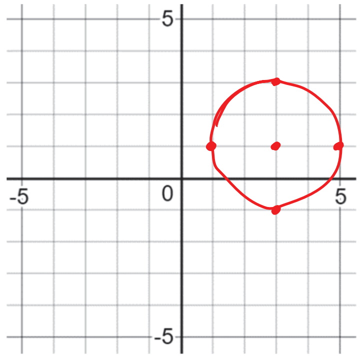
2 29

Example 2

Sketch the circle given by the equation

$x^2 - 6x + y^2 - 2y + 6 = 0$

and identify its center and radius.



center $(3, 1)$

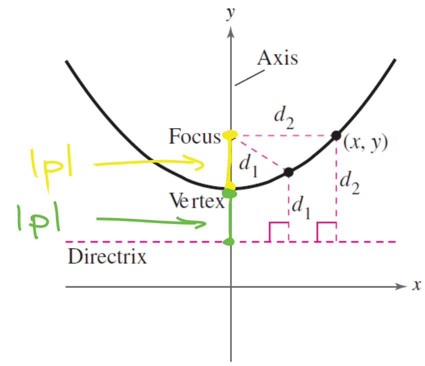
radius 2

$x^2 - 6x \underline{+9} + y^2 - 2y \underline{+1} = -6 \underline{+9} \underline{+1}$

$(x-3)^2 + (y-1)^2 = 4$

Definition of a Parabola

A **parabola** is the set of all points (x, y) in a plane that are equidistant from a fixed line, the **directrix**, and a fixed point, the **focus**, not on the line. The midpoint between the focus and the directrix is the **vertex**, and the line passing through the focus and the vertex is the **axis** of the parabola.



Standard Form of a Parabola

The **standard form of the equation of a parabola** with vertex at (h, k) is as follows.

Alg 2: $y = a(x-h)^2 + k$

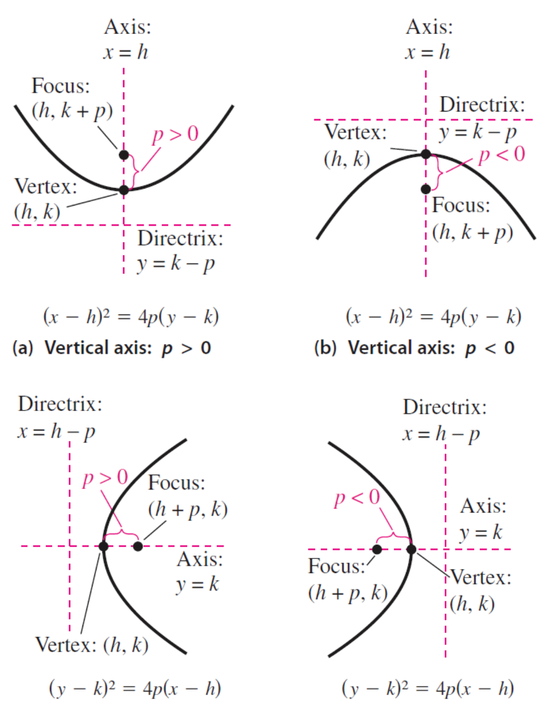
$(x-h)^2 = 4p(y-k)$ Vertical axis; directrix $y = k - p$

$(y-k)^2 = 4p(x-h)$ Horizontal axis; directrix $x = h - p$

The focus lies on the axis p units (*directed distance*) from the vertex. If the vertex is at the origin $(0, 0)$, the equation takes one of the following forms.

$x^2 = 4px$ Vertical axis

$y^2 = 4px$ Horizontal axis



$(x-h)^2 = 4p(y-k)$
(a) Vertical axis: $p > 0$

$(x-h)^2 = 4p(y-k)$
(b) Vertical axis: $p < 0$

$(y-k)^2 = 4p(x-h)$
(c) Horizontal axis: $p > 0$

$(y-k)^2 = 4p(x-h)$
(d) Horizontal axis: $p < 0$

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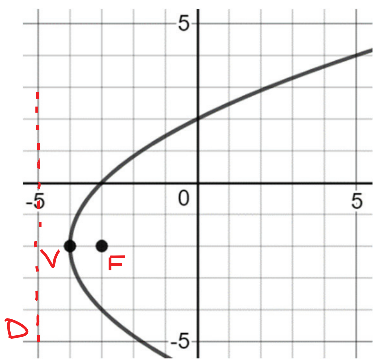
Example 3

Write the standard form of the equation of the parabola shown. Identify its vertex, focus, and directrix.



$(y-k)^2 = 4p(x-h)$

$(y+2)^2 = 4(x+4)$

vertex $(-4,-2)$ h, k

focus $(-3,-2)$ $p=1$

directrix $x=-5$

Example 4

Write the standard form of the parabola $y = \frac{1}{2}x^2 - x - \frac{1}{2}$.

Identify its vertex, focus, and directrix.

$2y = x^2 - 2x - 1$

$2y + 1 \underline{+1} = x^2 - 2x \underline{+1}$

$2y + 2 = (x-1)^2$

$2(y+1) = (x-1)^2$

V $(1,-1)$

F $(1,-1/2)$

D $y = -3/2$

$4p = 2$

$p = \frac{2}{4} = \frac{1}{2}$