Name ____________________

Date ______________ Pd ______

Definition of Geometric Sequence

A sequence is **geometric** if the ratios of consecutive terms are the same. So, the sequence $a_1,\ a_2,\ a_3,\ a_4,\ \ldots,\ a_n,\ \ldots$ is geometric if there is a number r such that

$$\frac{a_2}{a_1}=\frac{a_3}{a_2}=\frac{a_4}{a_3}=\cdots=r,\quad r\neq 0.$$

The number r is the **common ratio** of the sequence.

Example 1

Find the common ratio of the following sequences.

(a) $a_n = 4(3^n)$

(b) $a_n = \left(-\frac{1}{3}\right)^n$

Example 2

Find two formulas for the nth term of the geometric sequence $5,\ 15,\ 45,\ \ldots$. Assume n begins with 0 or 1.

Example 3

The fourth term of a geometric sequence is 125, and the 10th term is $125/64$. Find the 14th term. (Assume that the terms of the sequence are positive.)

The Sum of a Finite Geometric Sequence

The sum of the finite geometric sequence

$$a_1,\ a_1r,\ a_1r^2,\ a_1r^3,\ a_1r^4,\ \ldots,\ a_1r^{n-1}$$

with common ratio $r \neq 1$ is given by

$$S_n = \sum_{i=1}^{n} a_1 r^{i-1} = \frac{a_1(1-r^n)}{1-r}.$$

Example 4

Find the sum $\sum_{n=1}^{12} 4(0.3)^n$.

Geometric Series

The sum of the terms of an infinite geometric sequence is called an **infinite geometric series** or simply a **geometric series**.

The formula for the sum of a *finite geometric sequence* can, depending on the value of r, be extended to produce a formula for the sum of an *infinite geometric series*. Specifically, if the common ratio r has the property that $|r| < 1$, it can be shown that r^n becomes arbitrarily close to zero as n increases without bound. Consequently,

$$a_1\left(\frac{1-r^n}{1-r}\right) \to a_1\left(\frac{1-0}{1-r}\right) \text{ as } n \to \infty.$$

The Sum of an Infinite Geometric Series

If $|r| < 1$, then the infinite geometric series

$$a_1 + a_1r + a_1r^2 + a_1r^3 + \cdots + a_1r^{n-1} + \cdots$$

has the sum

$$S = \sum_{i=0}^{\infty} a_1 r^i = \frac{a_1}{1-r}.$$

Note that if $|r| \geq 1$, the series does not have a sum.

Example 5

Find the sum $\sum_{n=1}^{\infty} 4(0.6)^{n-1}$.

Example 6

A deposit of $50 is made on the first day of each month in a savings account that pays 6% compounded monthly. What is the balance at the end of 2 years?

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§8.3 Geometric Sequences and Series HOMEWORK Date ____________ Pd ______

In Exercises 1-2, write the first five terms of the geometric sequence. Determine whether or not the sequence is geometric. If it is, find the common ratio. (Assume n begins with 1.)

1. $1,\ -2,\ 4,\ -8,\ \ldots$

2. $1,\ \frac{1}{2},\ \frac{1}{3},\ \frac{1}{4},\ \ldots$

In Exercises 3-4, find two formulas for a_n for the geometric sequence. Assume n begins with 0 or 1.

3. $5,\ 10,\ 20,\ 40,\ \ldots$

4. $9,\ -6,\ 4,\ -8/3,\ \ldots$

In Exercises 5-6, write the first five terms of the geometric sequence. Assume n begins with 1.

5. $a_1 = 8,\ \ r = -3/4$

6. $a_3 = 16/3,\ \ a_5 = 64/27$
(Assume that the terms of the sequence are positive.)

In Exercises 7-8, find the sum of the finite geometric sequence.

7. $\sum_{i=1}^{6} 32\left(\frac{1}{4}\right)^{i-1}$

8. $\sum_{n=0}^{5} 300(1.06)^n$

In Exercises 9-10, find the sum of the infinite geometric series, if possible. If not possible, explain why.

9. $\sum_{n=0}^{\infty} 3(-0.9)^n$

10. $8+6+\frac{9}{2}+\frac{27}{8}+\cdots$

11. You have been hired at a company and the administration offers you two salary options.

Option 1: a starting salary of \$32,000 for the first year with salary increases of 2.5% each year for four years and then a reevaluation of performance

Option 2: a starting salary of \$32,500 for the first year with salary increases of 2% per year for four years and then a reevaluation of performance

(a) Which option do you choose if you want to make the greater cumulative amount for the five-year period? Explain your reasoning.

(b) Which option do you choose if you want to make the greater amount the year prior to reevaluation? Explain your reasoning.