

Definition of Arithmetic Sequence

A sequence is **arithmetic** if the differences between consecutive terms are the same. So, the sequence

$$a_1, a_2, a_3, a_4, \dots, a_n, \dots$$

is arithmetic if there is a number d such that

$$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \dots = d.$$

The number d is the **common difference** of the arithmetic sequence.

Example 1

Find the common difference of the following sequences.

(a) $a_n = 7 - 5n$

(b) $a_n = \frac{1}{4}(n + 3)$

Example 2

Find two formulas for the n th term of the arithmetic sequence whose common difference is 3 and whose first term is 2. Assume n begins with 0 or 1.

Example 3

The fourth term of an arithmetic sequence is 20, and the 13th term is 65. Write the first several terms of this sequence.

The Sum of a Finite Arithmetic Sequence

The sum of a finite arithmetic sequence with n terms is given by

$$S_n = \frac{n}{2}(a_1 + a_n).$$

Example 4

Find each sum.

(a) $1+3+5+7+9+11+13+15+17+19$

(b) Sum of the integers from 1 to 100

The sum of the first n terms of an arithmetic sequence is called the **n th partial sum**.

Example 5

Find the 50th partial sum of the arithmetic sequence
5, 16, 27, 38, 49,

Example 6

An auditorium has 30 rows of seats. There are 20 seats in the first row, 21 seats in the second row, 22 seats in the third row, and so on. How many seats are there altogether?

In Exercises 1-2, write the first five terms of the sequence. Determine whether or not the sequence is arithmetic. If it is, find the common difference. (Assume n begins with 1.)

1. $a_n = 3 + 2(-1)^n$

2. $a_n = 3 - 4(n + 6)$

In Exercises 3-4, find a formula for a_n for the arithmetic sequence.

3. $a_1 = 100, d = -8$

4. $a_3 = 94, a_6 = 85$

In Exercises 5-6, write the first five terms of the arithmetic sequence. Assume n begins with 1.

5. $a_1 = -10, d = -12$

6. $a_4 = 16, a_{10} = 46$

In Exercises 7-8, find the sum of the finite arithmetic sequence.

7. $-1 + (-3) + (-5) + (-7) + (-9)$

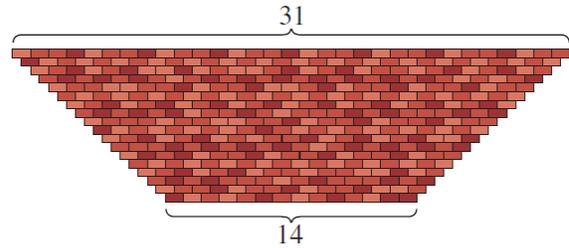
8. Sum of the first 50 positive odd integers

In Exercises 9-10, find the indicated n th partial sum of the arithmetic sequence.

9. $-6, -2, 2, 6, \dots, n = 50$

10. $0.5, 1.3, 2.1, 2.9, \dots, n = 10$

11. A brick patio has the approximate shape of a trapezoid, as shown in the figure. The patio has 18 rows of bricks. The first row has 14 bricks and the 18th row has 31 bricks. How many bricks are in the patio?



12. A small hardware store makes a profit of \$20,000 during its first year. The store owner sets a goal of increasing profits by \$5000 each year for 4 years. Assuming that this goal is met, find the total profit during the first 5 years of business.