Name ____________________

Date ______________ Pd ______

Definition of Sequence

An **infinite sequence** is a function whose domain is the set of positive integers. The function values

$$a_1,\ a_2,\ a_3,\ a_4,\ \ldots,\ a_n,\ \ldots$$

are the **terms** of the sequence. If the domain of a function consists of the first n positive integers only, the sequence is a **finite sequence.**

On occasion, it is convenient to begin subscripting a sequence with 0 instead of 1 so that the terms of the sequence become $a_0,\ a_1,\ a_2,\ a_3,\ \ldots$. The domain of the function is the set of nonnegative integers.

Example 1

Write the first five terms of each sequence.

(a) $a_n = 3n - 2$

1, 4, 7, 10, 13

(b) $a_n = \dfrac{(-1)^n}{2n-1}$ ← alternate

$\frac{-1}{1}$, $\frac{1}{3}$, $\frac{-1}{5}$

-1, $\frac{1}{3}$, $-\frac{1}{5}$, $\frac{1}{7}$, $-\frac{1}{9}$

Simply listing the first few terms is not sufficient to define a unique sequence—the nth term *must be given*. To see this, consider the following sequences, both of which have the same first three terms.

$$\frac{1}{2},\ \frac{1}{4},\ \frac{1}{8},\ \frac{1}{16},\ \ldots,\ \frac{1}{2^n},\ \ldots$$

$$\frac{1}{2},\ \frac{1}{4},\ \frac{1}{8},\ \frac{1}{15},\ \ldots,\ \frac{6}{(n+1)(n^2-n+6)},\ \ldots$$

Example 2

Write an expression for the apparent nth term (a_n) of each sequence.

(a) 1, 3, 5, 7, …

$a_n = 2n - 1$

(b) 2, 5, 10, 17, …

$a_n = n^2 + 1$

Definition of Factorial

If n is a positive integer, ***n* factorial** is defined as

$$n! = 1\cdot 2\cdot 3\cdot 4\cdots(n-1)\cdot n.$$

As a special case, zero factorial is defined as $0! = 1$.

Example 3

Write the first five terms of the sequence given by $a_n = \dfrac{2^n}{n!}$. Begin with $n = 0$.

$\frac{2^0}{0!}$, $\frac{2^1}{1!}$, $\frac{2^2}{2!}$, $\frac{2^3}{3!}$, $\frac{2^4}{4!}$

$\frac{1}{1}$ $\frac{2}{1}$ $\frac{4}{2}$ $\frac{8}{6}$ $\frac{16}{24}$

1, 2, 2, $\frac{4}{3}$, $\frac{2}{3}$

Example 4

(a) Simplify: $\dfrac{(2!)(8!)}{(3!)(5!)} = \dfrac{\cancel{(2!)}(8\cdot 7\cdot 6\cdot \cancel{5!})}{(3\cdot \cancel{2!})(\cancel{5!})}$

$= \dfrac{8\cdot 7\cdot \cancel{6}^{2}}{\cancel{3}}$

$= 112$

(b) Simplify: $\dfrac{n!}{(n-2)!} = \dfrac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}}$

$= n(n-1)$

$= n^2 - n$

Definition of Summation Notation

The sum of the first n terms of a sequence is represented by

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + a_4 + \cdots + a_n$$

where i is called the **index of summation**, n is the **upper limit of summation**, and 1 is the **lower limit of summation**.

Example 5

(a) Evaluate: $\sum_{i=1}^{5} 3i = 3+6+9+12+15$

$= 45$

(b) Evaluate: $\sum_{k=3}^{6} (1+k^2) = 10+17+26+37$

$= 90$

Properties of Sums

1. $\sum_{i=1}^{n} c = cn$, c is a constant.
2. $\sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i$, c is a constant.
3. $\sum_{i=1}^{n} (a_i \pm b_i) = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$

Definition of Series

Consider the infinite series $a_1,\ a_2,\ a_3,\ \ldots,\ a_i,\ \ldots$

1. The sum of the first n terms of the sequence is called a **finite series** or the **partial sum** of the sequence and is denoted by

$$a_1 + a_2 + a_3 + \cdots + a_n = \sum_{i=1}^{n} a_i.$$

2. The sum of all the terms of the infinite sequence is called an **infinite series** and is denoted by

$$a_1 + a_2 + a_3 + \cdots + a_n + \cdots = \sum_{i=1}^{\infty} a_i.$$

Example 6

For the series $\sum_{i=1}^{\infty} \dfrac{3}{10^i}$, find:

(a) the third partial sum.

$\sum_{i=1}^{3} \dfrac{3}{10^i} = 0.3 + 0.03 + 0.003$

$= 0.333$

(b) the sum.

$\sum_{i=1}^{\infty} \dfrac{3}{10^i} = 0.3 + 0.03 + 0.003 + 0.0003 + \cdots$

$= 0.\overline{3}$