Trigonometry/Precalculus
Name ______________________

§8.1 Sequences and Series
Date ______________ Pd ______

**Definition of Sequence**

An **infinite sequence** is a function whose domain is the set of positive integers. The function values

$$a_1,\ a_2,\ a_3,\ a_4,\ \ldots,\ a_n,\ \ldots$$

are the **terms** of the sequence. If the domain of a function consists of the first $n$ positive integers only, the sequence is a **finite sequence.**

On occasion, it is convenient to begin subscripting a sequence with $0$ instead of $1$ so that the terms of the sequence become $a_0,\ a_1,\ a_2,\ a_3,\ \ldots$. The domain of the function is the set of nonnegative integers.

**Example 1**

Write the first five terms of each sequence.

(a) $a_n = 3n - 2$

(b) $a_n = \dfrac{(-1)^n}{2n-1}$

Simply listing the first few terms is not sufficient to define a unique sequence—the $n$th term *must be given*. To see this, consider the following sequences, both of which have the same first three terms.

$$\frac{1}{2},\ \frac{1}{4},\ \frac{1}{8},\ \frac{1}{16},\ \ldots,\ \frac{1}{2^n},\ \ldots$$

$$\frac{1}{2},\ \frac{1}{4},\ \frac{1}{8},\ \frac{1}{15},\ \ldots,\ \frac{6}{(n+1)(n^2-n+6)},\ \ldots$$

**Example 2**

Write an expression for the apparent $n$th term $(a_n)$ of each sequence.

(a) $1,\ 3,\ 5,\ 7,\ \ldots$

(b) $2,\ 5,\ 10,\ 17,\ \ldots$

**Definition of Factorial**

If $n$ is a positive integer, ***n* factorial** is defined as

$$n! = 1 \cdot 2 \cdot 3 \cdot 4 \cdots (n-1) \cdot n.$$

As a special case, zero factorial is defined as $0! = 1$.

**Example 3**

Write the first five terms of the sequence given by $a_n = \dfrac{2^n}{n!}$. Begin with $n = 0$.

**Example 4**

(a) Simplify: $\dfrac{(2!)(8!)}{(3!)(5!)}$

(b) Simplify: $\dfrac{n!}{(n-2)!}$

**Definition of Summation Notation**

The sum of the first $n$ terms of a sequence is represented by

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + a_4 + \cdots + a_n$$

where $i$ is called the **index of summation**, $n$ is the **upper limit of summation**, and $1$ is the **lower limit of summation**.

**Example 5**

(a) Evaluate: $\sum_{i=1}^{5} 3i$

(b) Evaluate: $\sum_{k=3}^{6} (1+k^2)$

**Properties of Sums**

1. $\sum_{i=1}^{n} c = cn$, $c$ is a constant.
2. $\sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i$, $c$ is a constant.
3. $\sum_{i=1}^{n} (a_i \pm b_i) = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$

**Definition of Series**

Consider the infinite series $a_1,\ a_2,\ a_3,\ \ldots,\ a_i,\ \ldots$

1. The sum of the first $n$ terms of the sequence is called a **finite series** or the **partial sum** of the sequence and is denoted by

$$a_1 + a_2 + a_3 + \cdots + a_n = \sum_{i=1}^{n} a_i .$$

2. The sum of all the terms of the infinite sequence is called an **infinite series** and is denoted by

$$a_1 + a_2 + a_3 + \cdots + a_n + \cdots = \sum_{i=1}^{\infty} a_i .$$

**Example 6**

For the series $\sum_{i=1}^{\infty} \dfrac{3}{10^i}$, find:

(a) the third partial sum.

(b) the sum.

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In Exercises 1-2, write the first five terms of the sequence. Assume $n$ begins with 1.

1. $a_n = 2n + 5$

2. $a_n = (-1)^n \left( \dfrac{n}{n+1} \right)$

In Exercises 3-4, write two expressions for the *apparent* $n$th term of the sequence. Assume $n$ begins with 0 or 1.

3. $\dfrac{2}{1}, \dfrac{3}{3}, \dfrac{4}{5}, \dfrac{5}{7}, \dfrac{6}{9}, \ldots$

4. $1, -2, \dfrac{4}{2}, -\dfrac{8}{6}, \dfrac{16}{24}, -\dfrac{32}{120}, \ldots$

In Exercises 5-6, simplify the factorial expression.

5. $\dfrac{(10!)(3!)}{(4!)(6!)}$

6. $\dfrac{(2n-1)!}{(2n+1)!}$

In Exercises 7-8, find the sum.

7. $\sum_{i=0}^{4} i^2$

8. $\sum_{k=0}^{3} \frac{1}{k^2+1}$

In Exercises 9-10, use sigma notation to write the sum.

9. $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots + \frac{1}{19^2} - \frac{1}{20^2}$

10. $\frac{1}{1} + \frac{3}{2} + \frac{7}{6} + \frac{15}{24} + \frac{31}{120}$

In Exercises 11-12, find (a) the fourth partial sum and (b) the sum of the infinite series.

11. $\sum_{i=1}^{\infty} 6\left(\frac{1}{10}\right)^i$

12. $\sum_{k=1}^{\infty} 4\left(\frac{1}{10}\right)^k$