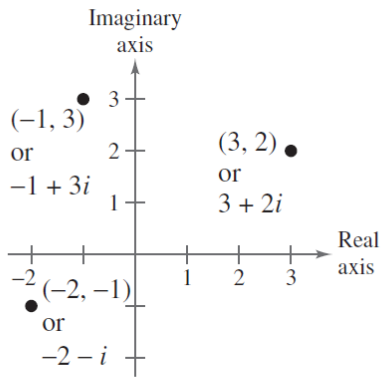
Name ____________________

Date ______________ Pd ______

The Complex Plane

You can represent a complex number $z = a + bi$ as the point (a, b) in a coordinate plane (the complex plane). The horizontal axis is called the real axis and the vertical axis is called the imaginary axis, as shown in the figure.



The **absolute value of a complex number** $a + bi$ is defined as the distance between the origin $(0, 0)$ and the point (a, b).

> **Definition of the Absolute Value of a Complex Number**
>
> The **absolute value** of the complex number $z = a + bi$ is given by
>
> $$|a + bi| = \sqrt{a^2 + b^2}.$$

If the complex number $a + bi$ is a real number (that is, if $b = 0$), then this definition agrees with that given for the absolute value of a real number

$$|a + 0i| = \sqrt{a^2 + 0^2} = |a|.$$

Example 1

Find the absolute value of $-4 + 4i$.

Trigonometric Form of a Complex Number

In Algebra II, you learned how to add, subtract, and multiply complex numbers. To work effectively with *powers* and *roots* of complex numbers, it is helpful to write complex numbers in trigonometric form. In the figure, consider the nonzero complex number $a + bi$. By letting θ be the angle from the positive real axis (measured counterclockwise) to the line segment connecting the origin and the point (a, b), you can write

$$a = r\cos(\theta) \text{ and } b = r\sin(\theta)$$

where $r = \sqrt{a^2 + b^2}$. Consequently, you have

$$a + bi = (r\cos(\theta)) + (r\sin(\theta))i$$

from which you can obtain the **trigonometric form of a complex number**.

> **Trigonometric Form of a Complex Number**
>
> The **trigonometric form** of the complex number $z = a + bi$ is given by
>
> $$z = r(\cos(\theta) + i\sin(\theta))$$
>
> where $a = r\cos(\theta)$, $b = r\sin(\theta)$, $r = \sqrt{a^2 + b^2}$, and $\tan(\theta) = b/a$. The number r is the **modulus** of z, and θ is called an **argument** of z.

The expression $\cos(\theta) + i\sin(\theta)$ can also be shortened as $\text{cis}(\theta)$. As such, we can also write $z = r\,\text{cis}(\theta)$.

The trigonometric form of a complex number is also called the *polar* form. Because there are infinitely many choices for θ, the trigonometric form of a complex number is not unique. Normally, θ is restricted to the interval $0 \le \theta < 2\pi$, although on occasion it is convenient to use $\theta < 0$.

Example 2

Find the trigonometric form of the complex number $\sqrt{3} + i$.

Example 3

Find the standard form of the number $2(\cos(120°)+i\sin(120°))$.

Multiplication and Division of Complex Numbers

The trigonometric form adapts nicely to multiplication and division of complex numbers. Suppose you are given two complex numbers

$$z_1 = r_1(\cos(\theta_1)+i\sin(\theta_1)) \text{ and } z_2 = r_2(\cos(\theta_2)+i\sin(\theta_2)).$$

The product of z_1 and z_2 is

$$\begin{aligned} z_1 z_2 &= r_1 r_2(\cos(\theta_1)+i\sin(\theta_1))(\cos(\theta_2)+i\sin(\theta_2)) \\ &= r_1 r_2\left[(\cos\theta_1\theta_2 - \sin\theta_1\theta_2)+i(\sin\theta_1\cos\theta_2+\cos\theta_1\sin\theta_2)\right]. \end{aligned}$$

Using the sum and difference formulas for cosine and sine, you can write this equation as

$$z_1 z_2 = r_1 r_2[\cos(\theta_1+\theta_2)+i\sin(\theta_1+\theta_2)].$$

This establishes the first part of the following rule. The second part can be verified in a similar manner.

Product and Quotient of Two Complex Numbers

Let $z_1 = r_1(\cos(\theta_1)+i\sin(\theta_1))$ and $z_2 = r_2(\cos(\theta_2)+i\sin(\theta_2))$ be complex numbers.

$$z_1 z_2 = r_1 r_2[\cos(\theta_1+\theta_2)+i\sin(\theta_1+\theta_2)]$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1-\theta_2)+i\sin(\theta_1-\theta_2)],\ z_2 \neq 0$$

Note that this rule says that to *multiply* two complex numbers you multiply moduli and add arguments, whereas to *divide* two complex numbers you divide moduli and subtract arguments.

Example 4

Evaluate.

$$\left[3\left(\cos\left(\frac{\pi}{3}\right)+i\sin\left(\frac{\pi}{3}\right)\right)\right]\left[4\left(\cos\left(\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{6}\right)\right)\right]$$

Example 5

Evaluate.

$$\frac{\cos(50°)+i\sin(50°)}{\cos(20°)+i\sin(20°)}$$

Trigonometry/Precalculus

§6.5a Trigonometric Form of a Complex Number HOMEWORK

Name ______________________

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In Exercises 1-2, find the absolute value.

1. $-5-12i$

2. $3+6i$

In Exercises 3-4, find the trigonometric form of the number.

3. $5-5i$

4. $-1-\sqrt{3}\,i$

In Exercises 5-6, find the standard form of the number.

5. $\frac{3}{2}\left(\cos(330°)+i\sin(330°)\right)$

6. $8\left(\cos\left(\frac{5\pi}{6}\right)+i\sin\left(\frac{5\pi}{6}\right)\right)$

In Exercises 7-8, evaluate.

7. $\left[\frac{3}{2}\left(\cos\left(\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{6}\right)\right)\right]\left[6\left(\cos\left(\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{4}\right)\right)\right]$

8. $\dfrac{12\left(\cos\left(\frac{7\pi}{4}\right)+i\sin\left(\frac{7\pi}{4}\right)\right)}{4\left(\cos(\pi)+i\sin(\pi)\right)}$

In Exercises 9-10, rewrite each complex number in trigonometric form, then evaluate.

9. $(2-2i)(1+i)$

10. $\dfrac{2i}{1-\sqrt{3}\,i}$