Trigonometry/Precalculus Name ____________

§6.2 Law of Cosines Date ____________ Pd ______

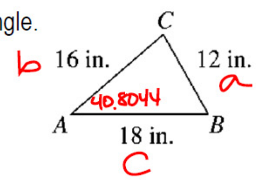
Two cases remain in the list of conditions needed to solve an oblique triangle—SSS and SAS. In these cases you can use the **Law of Cosines**.

Law of Cosines

Standard Form	*Alternative Form*
$a^2 = b^2 + c^2 - 2bc\cos(A)$	$\cos(A) = \dfrac{b^2+c^2-a^2}{2bc}$
$b^2 = a^2 + c^2 - 2ac\cos(B)$	$\cos(B) = \dfrac{a^2+c^2-b^2}{2ac}$
$c^2 = a^2 + b^2 - 2ab\cos(C)$	$\cos(C) = \dfrac{a^2+b^2-c^2}{2ab}$

Example 1 (SSS)

Use the Law of Cosines to solve the triangle.



$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

$$\cos A = \frac{16^2+18^2-12^2}{2\cdot16\cdot18}$$

$$\cos A = \frac{109}{144} \Rightarrow A = \cos^{-1}\left(\frac{109}{144}\right)$$

$$A = 40.8044^\circ$$

$$\cos B = \frac{a^2+c^2-b^2}{2ac}$$

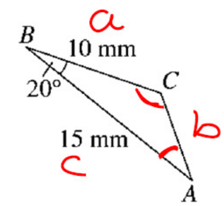
$$\cos B = \frac{12^2+18^2-16^2}{2\cdot12\cdot18} = \frac{53}{108}$$

$$B = \cos^{-1}\left(\frac{53}{108}\right) = 60.6107^\circ$$

$$C = 180 - A - B = 78.5848^\circ$$

Example 2 (SAS)

Use the Law of Cosines to solve the triangle.



$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$\sqrt{b^2} = \sqrt{10^2+15^2-2\cdot10\cdot15\cos 20^\circ}$$

$$b = 6.5644 \text{ or } 6.5645$$

$$\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{b^2+15^2-10^2}{2b\cdot15}$$

$$\cos A = 0.8535$$

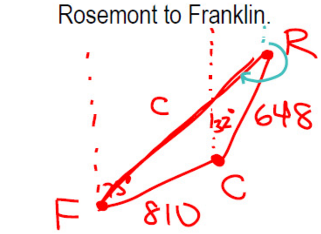
$$A = \cos^{-1}(0.8535) = 31.4005^\circ$$

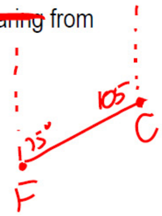
$$C = 180 - A - B$$

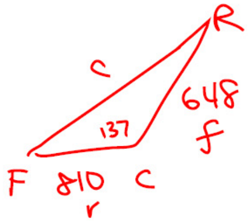
$$= 128.5994^\circ \text{ or } 128.5995^\circ$$

Example 3

A plane flies 810 miles from Franklin to Centerville with a bearing of 75°. Then it flies 648 miles from Centerville to Rosemont with a bearing of 32°. Find the straight-line distance ~~and bearing~~ from Rosemont to Franklin.







$$c^2 = f^2 + r^2 - 2fr\cos C$$

$$c = \sqrt{648^2 + 810^2 - 2\cdot 648 \cdot 810 \cos 137°}$$

$$= 1357.8475 \text{ mi}$$

Heron's Area Formula

Given any triangle with sides of lengths a, b, and c, the area of the triangle is given by

$$\text{area} = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = \dfrac{a+b+c}{2}$ is the semiperimeter.

Example 4

The Landau Building in Cambridge, Massachusetts has a triangular-shaped base. The lengths of the sides of the triangular base are 145 feet, 257 feet, and 290 feet. Find the area of the base of the building.

$$S = \frac{145+257+290}{2} = 346$$

$$A = \sqrt{346(346-145)(346-257)(346-290)}$$

$$= 18617.6600 \text{ ft}^2$$