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In Chapter 4 you looked at techniques for solving right triangles. In this section and the next, you will solve **oblique triangles**—triangles that have no right angles.

To solve an oblique triangle, you need to know the measure of at least one side and the measures of any two other parts of the triangle—two sides, two angles, or one angle and one side. This breaks down into the following four cases.

1. Two angles and any side (AAS or ASA)
2. Two sides and an angle opposite one of them (SSA)
3. Three sides (SSS)
4. Two sides and their included angle (SAS)

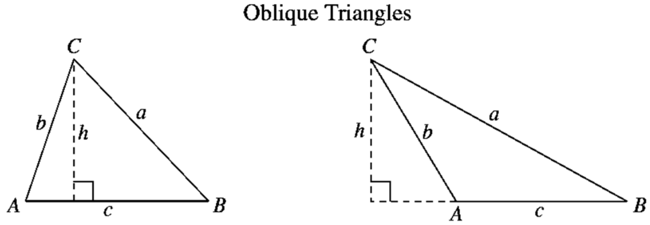
The first two cases can be solved using the **Law of Sines**, where as the last two cases require the Law of Cosines (see §6.2).

Law of Sines

If ABC is a triangle with sides a, b, and c, then

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)} \quad \text{OR} \quad \frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}.$$

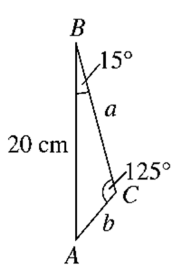
Oblique Triangles



A is acute. *A* is obtuse.

Example 1 (AAS)

Use the Law of Sines to solve the triangle.



Example 2 (ASA)

Use the Law of Sines to solve the triangle.

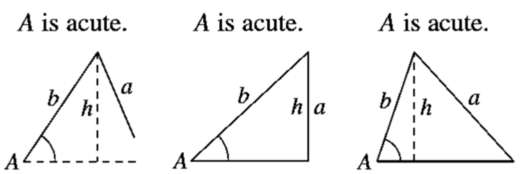
$A = 20°, \; B = 153°, \; c = 2.5$

The Ambiguous Case (SSA)

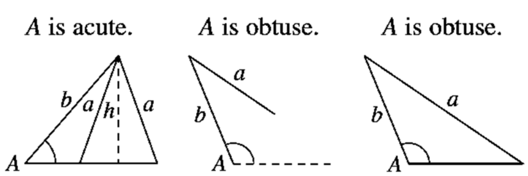
Two angles and one side determine a unique triangle. However, if two sides and one opposite angle are given, three possible situations can occur: (1) no such triangle exists, (2) one such triangle exists, or (3) two distinct triangles satisfy the conditions.

The Ambiguous Case (SSA)

Consider a triangle in which you are given a, b, and A $(h = b\sin(A))$.



Necessary condition	$a < h$	$a = h$	$a \geq b$
Possible triangles	None	One	One



Necessary condition	$h < a < b$	$a \leq b$	$a > b$
Possible triangles	Two	None	One

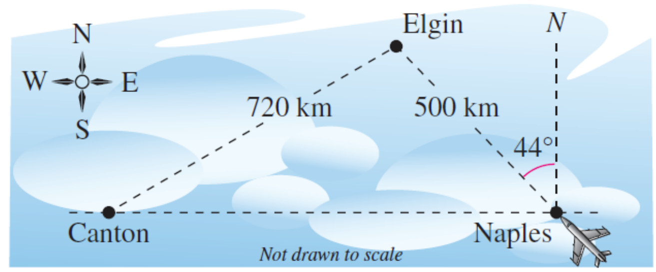
Example 3 (SSA No Solution Case)

Use the Law of Sines to solve the triangle.

$A = 76°,\ \ a = 18,\ \ b = 20$

Example 4 (SSA One Solution Case)

A plane flies 500 kilometers with a bearing of $316°$ from Naples to Elgin. The plane then flies 720 km from Elgin to Canton. Find the bearing of the flight from Elgin to Canton.



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Example 5 (SSA Two Solution Case)

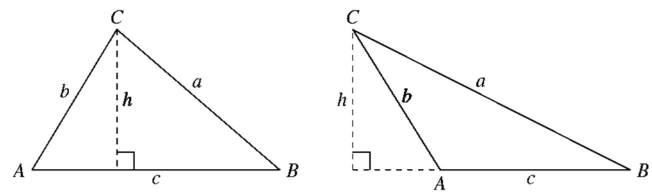
Use the Law of Sines to solve the triangle.
If two solutions exist, find both.

$A = 58°, \ a = 11.4, \ b = 12.8$

Area of an Oblique Triangle

The area of any triangle is one-half the product of the lengths of two sides times the sine of their included angle. That is,

$$\text{area} = \frac{1}{2}bc\sin(A) = \frac{1}{2}ab\sin(C) = \frac{1}{2}ac\sin(B).$$



Example 6

Find the area of the triangle having the indicated angle and sides.

$C = 110°, \ a = 6, \ b = 10$

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§6.1 Law of Sines HOMEWORK Date ____________ Pd ____

In Exercises 1-4, use the Law of Sines to solve the triangle.
If two solutions exist, find both.

1. $A = 102.4°$, $C = 16.7°$, $a = 21.6$

2. $A = 36°$, $a = 8$, $b = 5$

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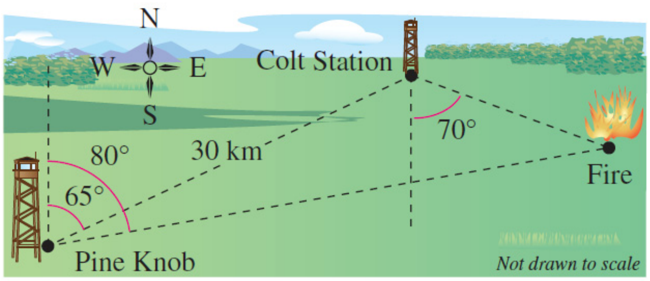
3. $A = 60°, \; a = 9, \; c = 10$

4. $A = 110°, \; a = 125, \; b = 200$

5. Find the area of the triangle having the indicated angle and sides.

   $B = 130°, \; a = 92, \; c = 30$

6. The bearing from the Pine Knob fire tower to the Colt Station fire tower is N 65° E, and the two towers are 30 kilometers apart. A fire spotted by rangers in each tower has a bearing of N 80° E from Pine Knob and S 70° E from Colt Station. Find the distance of the fire from each tower.



7. A 10-meter telephone pole casts a 17-meter shadow directly down a slope when the angle of elevation of the sun is 42°. Find θ, the angle of elevation of the ground.