Trigonometry/Precalculus

§5.5 Multiple-Angle Formulas

Name ____________________

Date ______________ Pd ______

Double-Angle Formulas

$\sin(2u) = 2\sin(u)\cos(u)$

$\tan(2u) = \dfrac{2\tan(u)}{1-\tan^2(u)}$

$\cos(2u) = \cos^2(u) - \sin^2(u)$

$= 2\cos^2(u) - 1$

$= 1 - 2\sin^2(u)$

Example 1

Solve the equation $\sin(2x) - \sin(x) = 0$ in the interval $[0, 2\pi)$.

FACTOR

$2\sin x\cos x - \sin x = 0$

$\sin x(2\cos x - 1) = 0$

$\sin x = 0$ $\quad\quad$ $2\cos x - 1 = 0$

$\cos x = \frac{1}{2}$

$x = 0, \pi$ $\quad\quad$ $x = \frac{\pi}{3}, \frac{5\pi}{3}$

$$\boxed{x = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}}$$

Example 2

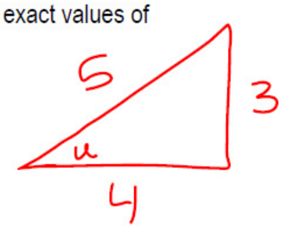
Rewrite the expression $8\sin(x)\cos(x)$.

$4 \cdot 2\sin x\cos x$

$$\boxed{4\sin(2x)}$$

Example 3

Given $\sin(u) = 3/5$, $0 < u < \pi/2$, find the exact values of $\sin(2u)$, $\cos(2u)$, and $\tan(2u)$.



$\sin(2u) = 2\sin u\cos u$

$= \frac{2}{1} \cdot \frac{3}{5} \cdot \frac{4}{5}$

$= \boxed{\frac{24}{25}}$

$\cos(2u) = \cos^2(u) - \sin^2(u)$

$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$= \frac{16}{25} - \frac{9}{25} = \boxed{\frac{7}{25}}$

$\tan(2u) = \dfrac{2\tan u}{1-\tan^2 u}$

$= \dfrac{2 \cdot \frac{3}{4}}{1 - \left(\frac{3}{4}\right)^2}$

$= \dfrac{\frac{3}{2}}{\frac{16}{16}1 - \frac{9}{16}} = \dfrac{\frac{3}{2} \cdot 16}{\frac{7}{16} \cdot 16} = \boxed{\frac{24}{7}}$

Half-Angle Formulas

$$\sin\left(\frac{u}{2}\right)=\pm\sqrt{\frac{1-\cos(u)}{2}} \qquad \cos\left(\frac{u}{2}\right)=\pm\sqrt{\frac{1+\cos(u)}{2}}$$

$$\tan\left(\frac{u}{2}\right)=\frac{1-\cos(u)}{\sin(u)}=\frac{\sin(u)}{1+\cos(u)}$$

The signs of $\sin(u/2)$ and $\cos(u/2)$ depend on the quadrant in which $u/2$ lies.

Example 4

Solve the equation $\sin\left(\frac{x}{2}\right)-\cos(x)=0$ in the interval $[0, 2\pi)$.

$+\cos x \quad +\cos x$

$$\sin^2\left(\frac{x}{2}\right)=\cos^2(x)$$

$$2\cdot\frac{1-\cos x}{2}=2\cos^2 x$$

$$1-\cos x=2\cos^2 x$$

$$\cos x=\frac{1}{2} \qquad \cos x=-1$$

$$x=\frac{\pi}{3}, \frac{5\pi}{3} \qquad x=\pi$$

$$1-c=2c^2$$

$$0=2c^2+c-1$$

$$0=(2c-1)(c+1)$$

$$2c-1=0 \qquad c+1=0$$

$$c=\frac{1}{2} \qquad c=-1$$

$$\boxed{x=\frac{\pi}{3}, \frac{5\pi}{3}}$$

CHECK YOUR SOLUTIONS!

Example 5

Determine the exact values of the sine, cosine, and tangent of the angle $15°.=30°/2$

$$\sin(15°)=\sqrt{\frac{1-\cos(30°)}{2}}=\sqrt{\frac{1-\frac{\sqrt{3}}{2}\cdot 2}{2\cdot 2}}=\sqrt{\frac{2-\sqrt{3}}{4}}=\boxed{\frac{\sqrt{2-\sqrt{3}}}{2}}$$

$$\cos(15°)=\sqrt{\frac{1+\cos(30°)}{2}}=\sqrt{\frac{1+\frac{\sqrt{3}}{2}\cdot 2}{2\cdot 2}}=\sqrt{\frac{2+\sqrt{3}}{4}}=\boxed{\frac{\sqrt{2+\sqrt{3}}}{2}}$$

$$\tan(15°)=\frac{1-\cos(30°)}{\sin(30°)}=\frac{1-\frac{\sqrt{3}}{2}\cdot 2}{\frac{1}{2}\cdot 2}=\boxed{2-\sqrt{3}}$$

Power-Reducing Formulas

$$\sin^2(u)=\frac{1-\cos(2u)}{2} \qquad \cos^2(u)=\frac{1+\cos(2u)}{2}$$

$$\tan^2(u)=\frac{1-\cos(2u)}{1+\cos(2u)}$$

Example 6

Rewrite the expression $\cos^4(x)$.

$$\cos^4(x)=\left(\cos^2 x\right)^2=\left(\frac{1+\cos 2x}{2}\right)^2=\frac{(1+\cos 2x)(1+\cos 2x)}{2^2}=\boxed{\frac{1+2\cos 2x+\cos^2(2x)}{4}}$$