

Double-Angle Formulas

$$\begin{aligned}\sin(2u) &= 2\sin(u)\cos(u) & \cos(2u) &= \cos^2(u) - \sin^2(u) \\ \tan(2u) &= \frac{2\tan(u)}{1 - \tan^2(u)} & &= 2\cos^2(u) - 1 \\ & & &= 1 - 2\sin^2(u)\end{aligned}$$

Example 1

Solve the equation $\sin(2x) - \sin(x) = 0$ in the interval $[0, 2\pi)$.

Example 2

Rewrite the expression $8\sin(x)\cos(x)$.

Example 3

Given $\sin(u) = 3/5$, $0 < u < \pi/2$, find the exact values of $\sin(2u)$, $\cos(2u)$, and $\tan(2u)$.

Half-Angle Formulas

$$\sin\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1-\cos(u)}{2}} \qquad \cos\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1+\cos(u)}{2}}$$

$$\tan\left(\frac{u}{2}\right) = \frac{1-\cos(u)}{\sin(u)} = \frac{\sin(u)}{1+\cos(u)}$$

The signs of $\sin(u/2)$ and $\cos(u/2)$ depend on the quadrant in which $u/2$ lies.

Example 4

Solve the equation $\sin\left(\frac{x}{2}\right) - \cos(x) = 0$ in the interval $[0, 2\pi)$.

Example 5

Determine the exact values of the sine, cosine, and tangent of the angle 15° .

Power-Reducing Formulas

$$\sin^2(u) = \frac{1-\cos(2u)}{2} \qquad \cos^2(u) = \frac{1+\cos(2u)}{2}$$

$$\tan^2(u) = \frac{1-\cos(2u)}{1+\cos(2u)}$$

Example 6

Rewrite the expression $\cos^4(x)$.

In Exercises 1-2, solve the equation in the interval $[0, 2\pi)$.

1. $\sin(2x)\sin(x) = \cos(x)$

2. $\cos(2x) - \cos(x) = 0$

3. Given $\cos(u) = -2/7$, $\pi/2 < u < \pi$, find the exact values of $\sin(2u)$, $\cos(2u)$, and $\tan(2u)$.

4. Rewrite the expression $6 - 12\sin^2(x)$.

5. Rewrite the expression $\sin^4(x)$.

6. Determine the exact values of the sine, cosine, and tangent of the angle 165° .

7. Given $\tan(u) = -8/5$, $3\pi/2 < u < 2\pi$, find the exact values of $\sin(u/2)$, $\cos(u/2)$, and $\tan(u/2)$.

8. Solve the equation $\cos\left(\frac{x}{2}\right) - \sin(x) = 0$ in the interval $[0, 2\pi)$.