Trigonometry/Precalculus

§5.4 Sum and Difference Formulas

Name _____

Date _____ Pd _____

Sum and Difference Formulas

$$\sin(u \pm v) = \sin(u)\cos(v) \pm \cos(u)\sin(v)$$
$$\cos(u \pm v) = \cos(u)\cos(v) \mp \sin(u)\sin(v)$$
$$\tan(u \pm v) = \frac{\tan(u) \pm \tan(v)}{1 \mp \tan(u)\tan(v)}$$

Example 1

Find the exact value of $\tan(105°)$.

$\tan(105°) = \tan(60° + 45°)$ (u + v)

$= \dfrac{\tan(60°) + \tan(45°)}{1 - \tan(60°)\tan(45°)}$

$= \dfrac{\sqrt{3} + 1}{1 - \sqrt{3} \cdot 1}$

$= \boxed{\dfrac{\sqrt{3} + 1}{1 - \sqrt{3}}}$

Example 2

Find the exact value of $\cos\left(\frac{7\pi}{12}\right)$.

$\cos\left(\frac{7\pi}{12}\right) = \cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$ (u + v)

$= \cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) - \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{3}\right)$

$= \frac{\sqrt{2}}{2} \cdot \frac{1}{2} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$

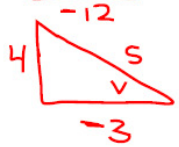
$= \boxed{\dfrac{\sqrt{2} - \sqrt{6}}{4}}$

Example 3

Given that $\sin(u) = 5/13$, $\cos(v) = -3/5$, and both u and v are in Quadrant II, find the exact value of $\sin(u + v)$.

$\sin(u+v) = \sin(u)\cos(v) + \cos(u)\sin(v)$

$= \frac{5}{13} \cdot \frac{-3}{5} + \frac{-12}{13} \cdot \frac{4}{5}$



$= \frac{-15 - 48}{65} = \boxed{-\frac{63}{65}}$

Example 4

Write the trigonometric expression $\sin(\underbrace{\arcsin(x)}_{u}+\underbrace{\arccos(x)}_{v})$ as an algebraic expression.

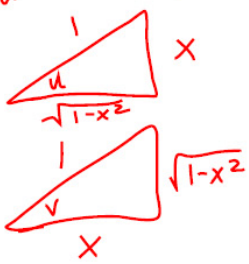
$\sin(\arcsin x+\arccos x)=$

$\sin(\arcsin x)\cos(\arccos x)+\cos(\arcsin x)\sin(\arccos x)$

$= x\cdot x+\sqrt{1-x^2}\cdot\sqrt{1-x^2}$

$= x^2+1-x^2$

$= \boxed{1}$



Example 5

Find the solution(s) of the equation in the interval $[0, 2\pi)$.

$$\sin\left(x+\frac{\pi}{3}\right)+\sin\left(x-\frac{\pi}{3}\right)=1$$

$\sin(x)\cos\left(\frac{\pi}{3}\right)+\cos(x)\sin\left(\frac{\pi}{3}\right)+\sin(x)\cos\left(\frac{\pi}{3}\right)-\cos(x)\sin\left(\frac{\pi}{3}\right)=1$

$2\sin(x)\cos\left(\frac{\pi}{3}\right)=1$

$2\sin(x)\cdot\frac{1}{2}=1$

$\sin(x)=1$

$\boxed{x=\frac{\pi}{2}}$