Trigonometry/Precalculus

Name _____

§5.3b Solving Trigonometric Equations

Date _____ Pd _____

Example 1

Find all solutions of the equation $\csc(x)+\cot(x)=1$ in the interval $[0, 2\pi)$ algebraically.

SQUARE BOTH SIDES

*CHECK

$-\cot(x)$ $-\cot(x)$

$\csc^2(x) = (1-\cot(x))^2$

$\csc^2(x) = (1-\cot(x))(1-\cot(x))$ FOIL!

$\csc^2(x) = \cot^2(x) - 2\cot(x) + 1$

$1+\cot^2(x) = \cot^2(x) - 2\cot(x) + 1$

$-1 \quad -\cot^2(x) \quad -\cot^2(x) \quad -1$

$\frac{0}{-2} = \frac{-2\cot(x)}{-2}$

$0 = \cot(x)$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$x=\frac{\pi}{2}$: $\csc\left(\frac{\pi}{2}\right)+\cot\left(\frac{\pi}{2}\right) = 1+0 = 1$

~~$x=\frac{3\pi}{2}$~~ : $\csc\left(\frac{3\pi}{2}\right)+\cot\left(\frac{3\pi}{2}\right) = -1+0 = -1$

$x = \frac{\pi}{2}$

$(1-c)(1-c) = 1-c-c+c^2$

$= c^2-2c+1$

Example 2

Solve the equation $\sin(2x) = -\frac{\sqrt{3}}{2}$ algebraically.

$\sin(\theta) = -\frac{\sqrt{3}}{2}$

$\theta = \frac{4\pi}{3}+2\pi k, \frac{5\pi}{3}+2\pi k, k\in\mathbb{Z}$

$\frac{2x}{2} = \frac{1}{2}\frac{4\pi}{3}+\frac{2\pi k}{2}, \frac{1}{2}\frac{5\pi}{3}+\frac{2\pi k}{2}, k\in\mathbb{Z}$ (k is an integer)

$x = \frac{2\pi}{3}+\pi k, \frac{5\pi}{6}+\pi k, k\in\mathbb{Z}$

Example 3

Find all solutions of the equation $\cos\left(\frac{x}{2}\right)=\frac{\sqrt{2}}{2}$ in the interval $[0, 2\pi)$ algebraically.

$\frac{0}{2} \le \frac{x}{2} < \frac{2\pi}{2}$

$0 \le \frac{x}{2} < \pi$

$0 \le \theta < \pi \qquad \cos(\theta)=\frac{\sqrt{2}}{2}$

$\theta = \frac{\pi}{4}$

$2 \cdot \frac{x}{2} = \frac{\pi}{4} \cdot 2$

$x = \frac{\pi}{2}$

Example 4

Find all solutions of the equation $\sec^2(x)+\tan(x)=3$ in the interval $[0, 2\pi)$ algebraically.

$1+\tan^2(x)+\tan(x)=3$

$-3 \qquad -3$

$\tan^2(x)+\tan(x)-2=0$

$t^2+t-2=0$

$(t+2)(t-1)=0$

$t+2=0 \qquad t-1=0$

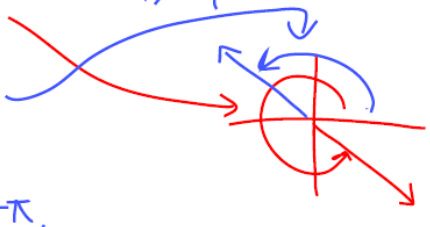
$-2 \ -2 \qquad +1 \ +1$

$t=-2 \qquad t=1$

$\tan(x)=-2 \qquad \tan(x)=1$

$x=\arctan(-2)+2\pi \qquad x=\frac{\pi}{4}, \frac{5\pi}{4}$

$x=\arctan(-2)+\pi$



$x=\frac{\pi}{4}, \arctan(-2)+\pi,$

$\frac{5\pi}{4}, \arctan(-2)+2\pi$

Example 5

Find all solutions of the equation $3\tan^2(x)+5\tan(x)-4=0$ in the interval $[-\pi/2, \pi/2]$ graphically.

graph — use zero feature

adjust window

$x = -1.1537$ or $-1.1538, 0.5335$

Trigonometry/Precalculus

Name _____

§5.3b Solving Trigonometric Equations HOMEWORK

Date _____ Pd _____

In Exercises 1-3, solve the equation algebraically.

1. $\sec(x)+\tan(x)=1$

$-\tan(x)$ $-\tan(x)$

$\sec^2(x) = (1-\tan(x))^2$

$\sec^2(x) = 1-2\tan(x)+\tan^2(x)$

$1+\tan^2(x) = 1-2\tan(x)+\tan^2(x)$

$-1\ -\tan^2(x)\ -1 \qquad -\tan^2(x)$

$\frac{0}{-2} = \frac{-2\tan(x)}{-2}$

$0 = \tan(x)$

$x = 0+2\pi k,\ \pi+2\pi k,\ k\in\mathbb{Z}$

OR $x = 0+\pi k,\ k\in\mathbb{Z}$

2. $2\sin^2(2x)=1$

$\frac{2}{2}\quad\frac{2}{2}$

$\sqrt{\sin^2(2x)} = \pm\sqrt{\frac{1}{2}}$

$\sin(2x) = \pm\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$

$\sin(2x) = \pm\frac{\sqrt{2}}{2}$

$\frac{2x}{2} = \frac{1}{2}\frac{\pi}{4}+\frac{2\pi k}{2},\ \frac{1}{2}\frac{3\pi}{4}+\frac{2\pi k}{2},\ \frac{1}{2}\frac{5\pi}{4}+\frac{2\pi k}{2},\ \frac{1}{2}\frac{7\pi}{4}+\frac{2\pi k}{2}$

$x = \frac{\pi}{8}+\pi k,\ \frac{3\pi}{8}+\pi k,\ \frac{5\pi}{8}+\pi k,\ \frac{7\pi}{8}+\pi k$

$k\in\mathbb{Z}$

OR $x = \frac{\pi}{8}+\frac{\pi}{4}k,\ k\in\mathbb{Z}$

3. $\tan\left(\frac{x}{3}\right)=1$

$\tan(\theta)=1$

$\theta = \frac{\pi}{4}+2\pi k,\ \frac{5\pi}{4}+2\pi k$

$\frac{x}{3} = 3\cdot\frac{\pi}{4}+3\cdot 2\pi k,\ 3\cdot\frac{5\pi}{4}+3\cdot 2\pi k$

$x = \frac{3\pi}{4}+6\pi k,\ \frac{15\pi}{4}+6\pi k,$

$k\in\mathbb{Z}$

In Exercises 4-6, find all solutions of the equation in the interval $[0, 2\pi)$ algebraically.

$2\cdot 0\le 2x<2\pi\cdot 2$

$0\le 2x<4\pi$

4. $\cos(2x)=-1$

$\cos(\theta)=-1$

$\theta = \pi,\ 3\pi$

$\frac{2x}{2} = \frac{\pi}{2},\ \frac{3\pi}{2}$

$x = \frac{\pi}{2},\ \frac{3\pi}{2}$

5. $\sec(4x) = 2$

$4 \cdot 0 \le x < 2\pi \cdot 4$

$0 \le 4x < 8\pi$

$\sec(\theta) = 2$

$\cos(\theta) = \frac{1}{2}$

$\theta = \left(\frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3},\right.$

$\left.\frac{13\pi}{3}, \frac{17\pi}{3}, \frac{19\pi}{3}, \frac{23\pi}{3}\right) \div 4$

$\frac{4x}{4}$

$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12},$

$\frac{19\pi}{12}, \frac{23\pi}{12}$

6. $\cos\left(\frac{x}{4}\right) = 0$

$\frac{0}{4} \le \frac{x}{4} < \frac{2\pi}{4}$

$0 \le \theta < \frac{\pi}{2}$

$\cos(\theta) = 0$

no solution

In Exercises 7-8, find all solutions of the equation in the interval $[-\pi/2, \pi/2]$ graphically.

7. $4\cos^2(x) - 2\sin(x) + 1 = 0$

$x = 1.1098$ or 1.1099

8. $2\sec^2(x) + \tan(x) = 6 = 0$

Whoops typo, I think that should be a $-$

$x = -1.0354$ or -1.0355, 0.8703