Trigonometry/Precalculus

§5.3a Solving Trigonometric Equations

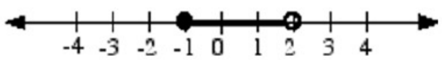
Name ____________________

Date ______________ Pd ______

Interval Notation (from Mathwords.com)

Interval notation is a notation for representing an interval as a pair of numbers. The numbers are the endpoints of the interval.

Parentheses and/or brackets are used to show whether the endpoints are included or excluded. For example, $[-1, 2)$ is the interval of real numbers between -1 and 2, including -1 and excluding 2.



Inequality Notation	*Interval Notation*
$1 < x < 4$	$(1, 4)$
$-3 \le x \le 8$	$[-3, 8]$
$2 \le x < 5$	$[2, 5)$
$x > 6$	$(6, \infty)$
$x \le 7$	$(-\infty, 7]$

Introduction

To solve a trigonometric equation, use standard algebraic techniques such as collecting like terms and factoring. Your preliminary goal is to $\boxed{\text{isolate the trigonometric function}}$ involved in the equation.

Example 1

Solve the equation $2\cos(x) + 1 = 0$ algebraically.

$-1 \quad -1$

$\frac{2\cos(x)}{2} = \frac{-1}{2}$

$\cos(x) = -\frac{1}{2}$

$2y + 1 = 0$

$-1 \quad -1$

$\frac{2y}{2} = \frac{-1}{2}$

$y = -\frac{1}{2}$

$x = \frac{2\pi}{3} + 2\pi k, \ \frac{4\pi}{3} + 2\pi k,$ k is an integer (or $k \in \mathbb{Z}$)

Example 2

Solve the equation $\csc^2(x) - 2 = 0$ algebraically.

$+2 \quad +2$

$\csc^2(x) = 2$

$\csc(x) = \pm\sqrt{2}$

$\sin(x) = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$

$y^2 - 2 = 0$

$+2 \quad +2$

$y^2 = 2$

$|y| = \sqrt{2}$

$y = \pm\sqrt{2}$

$x = \frac{\pi}{4} + 2\pi k, \ \frac{3\pi}{4} + 2\pi k, \ \frac{5\pi}{4} + 2\pi k, \ \frac{7\pi}{4} + 2\pi k,$

k is an integer (or $k \in \mathbb{Z}$)

Example 3

Find all solutions of the equation $3\tan^3(x) = \tan(x)$ in the interval $[0, 2\pi)$ algebraically.

$0 \le x < 2\pi$

$\tan(x) = 0 \qquad \tan(x) = \pm\frac{1}{\sqrt{3}}$

$x = 0, \pi \qquad x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$x = 0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}$

$3y^3 = y$

$-y \quad -y$

$3y^3 - y = 0$

$y(3y^2 - 1) = 0$

$y = 0 \qquad 3y^2 - 1 = 0$

$+1 \quad +1$

$\frac{3y^2}{3} = \frac{1}{3}$

$y^2 = \frac{1}{3}$

$y = \pm\frac{1}{\sqrt{3}}$

Example 4

Find all solutions of the equation $2\sin^2(x)+3\sin(x)+1=0$ in the interval $[0, 2\pi)$ algebraically.

$2y^2+3y+1=0$

$(2y+1)(y+1)=0$

$2y+1=0 \quad y+1=0$

$y=-\frac{1}{2} \quad y=-1$

$\sin(x)=-\frac{1}{2} \qquad \sin(x)=-1$

$x=\frac{7\pi}{6}, \frac{11\pi}{6} \qquad x=\frac{3\pi}{2}$

$x=\frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

Example 5

Find all solutions of the equation $2\sin^2(x)=\cos(x)+1$ in the interval $[0, 2\pi)$ algebraically.

pythag identity

$2(1-\cos^2 x) = \cos x + 1$

$2 - 2\cos^2 x = \cos x + 1$

$-2 + 2\cos^2 x \quad + 2\cos^2 x \quad -2$

$0 = 2\cos^2 x + \cos x - 1$

$0 = 2y^2+y-1$

$0=(2y-1)(y+1)$

$2y-1=0 \quad y+1=0$

$y=\frac{1}{2} \quad y=-1$

$\cos x = \frac{1}{2} \qquad \cos x = -1$

$x=\frac{\pi}{3}, \frac{5\pi}{3} \qquad x=\pi$

$x=\frac{\pi}{3}, \pi, \frac{5\pi}{3}$