Name ____________________

Date ____________ Pd ______

Introduction

In this section, you will study techniques for verifying trigonometric identities. In the next section, you will study techniques for solving trigonometric equations. The key to both verifying identities and solving equations is your ability to use the fundamental identities and the rules of algebra to rewrite trigonometric expressions.

Remember that a *conditional equation* is an equation that is true for only some of the values in its domain. For example, the conditional equation $\sin(x) = 0$ is true only for $x = n\pi$, where n is an integer. When you find these values, you are *solving* the equation.

On the other hand, an equation that is true for all real values in the domain of the variable is an *identity*. For example, the familiar equation $\sin^2(x) = 1 - \cos^2(x)$ is true for all real numbers x. So, it is an identity.

Verifying that a trigonometric equation is an identity is quite different from solving an equation. There is no well-defined set of rules to follow in verifying trigonometric identities, and the process is best learned by practice.

Guidelines for Verifying Trigonometric Identities

1. Work with one side of the equation at a time. It is often better to work with the more complicated side first.
2. Look for opportunities to factor an expression, add fractions, square a binomial, or create a monomial denominator.
3. Look for opportunities to use the fundamental identities. Note which functions are in the final expression you want. Sines and cosines pair up well, as do secants and tangents, and cosecants and cotangents.
4. If the preceding guidelines do not help, try converting all terms to sines and cosines.
5. Always try *something*. Even making an attempt that leads to a dead end provides insight.

Verifying trigonometric identities is a useful process if you need to convert a trigonometric expression into a form that is more useful algebraically. When you verify an identity, you cannot assume that the two sides of the equation are equal because you are trying to verify that they are equal. As a result, when verifying identities, you cannot use operations such as adding the same quantity to each side of the equation or cross multiplication.

Example 1

Verify the identity $\cos^2(\beta) - \sin^2(\beta) = 1 - 2\sin^2(\beta)$.

Example 2

Verify the identity $\dfrac{\cos(x) - \cos(y)}{\sin(x) + \sin(y)} + \dfrac{\sin(x) - \sin(y)}{\cos(x) + \cos(y)} = 0$.

Example 3

Verify the identity
$2\sec^2(x) - 2\sec^2(x)\sin^2(x) - \sin^2(x) - \cos^2(x) = 1$.

Example 4

Verify the identity $\dfrac{\cot(x)\tan(x)}{\sin(x)} = \csc(x)$.

Example 5

Verify the identity $\dfrac{\sin(\beta)}{1-\cos(\beta)} = \dfrac{1+\cos(\beta)}{\sin(\beta)}$.

Example 6

Verify the identity $\dfrac{\tan^3(\alpha)-1}{\tan(\alpha)-1} = \tan^2(\alpha) + \tan(\alpha) + 1$.

Trigonometry/Precalculus

Name ______________________

§5.2 Verifying Trigonometric Identities HOMEWORK

Date ______________ Pd ______

In Exercises 1-8, verify the identity.

1. $\dfrac{\csc^2(x)}{\cot(x)} = \csc(x)\sec(x)$

2. $\cos^2(\beta) - \sin^2(\beta) = 2\cos^2(\beta) - 1$

3. $\sec^6(x)\left(\sec(x)\tan(x)\right) - \sec^4(x)\left(\sec(x)\tan(x)\right) = \sec^5(x)\tan^3(x)$

4. $\dfrac{\tan(x) + \cot(y)}{\tan(x)\cot(y)} = \tan(y) + \cot(x)$

5. $\dfrac{1+\csc(\theta)}{\sec(\theta)}-\cot(\theta)=\cos(\theta)$

6. $\csc^4(x)-2\csc^2(x)+1=\cot^4(x)$

7. $\dfrac{\cot(\alpha)}{\csc(\alpha)-1}=\dfrac{\csc(\alpha)+1}{\cot(\alpha)}$

8. $\dfrac{\sin^3(\beta)+\cos^3(\beta)}{\sin(\beta)+\cos(\beta)}=1-\sin(\beta)\cos(\beta)$