Name ______________________
Date ______________ Pd ______

Introduction

In chapter 4, you studied the basic definitions, properties, graphs, and applications of the individual trigonometric functions. In this chapter, you will learn how to use the fundamental identities to do the following.

1. Evaluate trigonometric functions.
2. Simplify trigonometric expressions.
3. Develop additional trigonometric identities.
4. Solve trigonometric equations.

Fundamental Trigonometric Identities

Reciprocal Identities

$$\sin(u)=\frac{1}{\csc(u)} \qquad \cos(u)=\frac{1}{\sec(u)} \qquad \tan(u)=\frac{1}{\cot(u)}$$

$$\csc(u)=\frac{1}{\sin(u)} \qquad \sec(u)=\frac{1}{\cos(u)} \qquad \cot(u)=\frac{1}{\tan(u)}$$

Quotient Identities

$$\tan(u)=\frac{\sin(u)}{\cos(u)} \qquad \cot(u)=\frac{\cos(u)}{\sin(u)}$$

Pythagorean Identities

$$\sin^2(u)+\cos^2(u)=1 \qquad 1+\tan^2(u)=\sec^2(u)$$
$$1+\cot^2(u)=\csc^2(u)$$

Example 1 (Simplifying a Trigonometric Expression)

Use the fundamental identities to simplify the expression

$\sin(\theta)(\csc(\theta)-\sin(\theta))$

$$\sin(\theta)\left(\frac{1}{\sin(\theta)}-\sin(\theta)\right)$$

rewrite w/ reciprocal identities

multiply together

$$1-\sin^2(\theta)$$

$$\boxed{\cos^2(\theta)}$$

manipulate Pythag. ident.

$$\sin^2(\theta)+\cos^2(\theta)=1$$
$$-\sin^2(\theta) \qquad -\sin^2(\theta)$$
$$\cos^2(\theta)=1-\sin^2(\theta)$$

Example 2 (Verifying a Trigonometric Identity)

Verify the identity algebraically. Verify with a graphing calculator.

$\sin(\theta)+\cos(\theta)\cot(\theta)=\csc(\theta)$

$$\sin(\theta)+\cos(\theta)\cot(\theta)=\sin(\theta)+\cos(\theta)\cdot\frac{\cos(\theta)}{\sin(\theta)}$$

$$=\frac{\sin(\theta)\cdot\sin(\theta)}{\sin(\theta)\cdot 1}+\frac{\cos^2(\theta)}{\sin(\theta)}$$

make a common denominator

$$=\frac{\sin^2(\theta)+\cos^2(\theta)}{\sin(\theta)} \quad \star$$

Pythagorean identity

$$=\frac{1}{\sin(\theta)}$$

$$=\csc(\theta)$$

Example 3 (Verifying a Trigonometric Identity)

Verify the identity algebraically. Verify with a graphing calculator.

$\csc(\theta)\tan(\theta)=\sec(\theta)$

$$\csc(\theta)\tan(\theta)=\frac{1}{\sin(\theta)}\cdot\frac{\sin(\theta)}{\cos(\theta)}$$

$$=\frac{1}{\cos(\theta)}$$

$$=\sec(\theta)$$

Example 4 (Factoring a Trigonometric Expression)

Factor the expression $\cot^2(x) - \cot^2(x)\cos^2(x)$ and use the fundamental identities to simplify. Verify with a graphing calculator.

$\cot^2(x) - \cot^2(x)\cos^2(x)$

$\cot^2(x)\left(1 - \cos^2(x)\right)$ Pythag. ident.

$\cot^2(x) \cdot \sin^2(x)$

$\frac{\cos^2(x)}{\sin^2(x)} \cdot \sin^2(x)$

$\boxed{\cos^2(x)}$

Example 6 (Simplifying a Trigonometric Expression)

Perform the indicated operation and use the fundamental identities to simplify.

$\frac{\tan(x)}{1} \cdot \frac{\tan(x)}{\tan(x)} - \frac{\sec^2(x)}{\tan(x)}$

common denominator

$\frac{\tan^2(x) - \sec^2(x)}{\tan(x)}$

Pythag. ident.

$1 + \tan^2(x) = \sec^2(x)$
$-1 \quad - \sec^2(x) \quad - \sec^2(x) \quad -1$
$\tan^2(x) - \sec^2(x) = -1$

$\frac{-1}{\tan(x)}$

$\boxed{-\cot(x)}$

Example 5 (Factoring a Trigonometric Expression)

Factor the expression $\sin^4(x) - \cos^4(x)$ and use the fundamental identities to simplify. Verify with a graphing calculator.

$(\sin^2(x))^2 = \sin^4(x)$ $\quad (\cos^2(x))^2 = \cos^4(x)$

$(\sin^2(x))^2 - (\cos^2(x))^2$ $\quad a^2 - b^2 = (a+b)(a-b)$ difference of squares

$(\sin^2(x) + \cos^2(x))(\sin^2(x) - \cos^2(x))$

$1\,(\sin^2(x) - \cos^2(x))$

$\boxed{\sin^2(x) - \cos^2(x)}$

Example 7 (Rewriting a Trigonometric Expression)

Rewrite the expression $\frac{\sin^2(y)}{1 - \cos(y)}$ so that it is not in fractional form.

multiply by conjugate

$a - b \quad a + b$
$a + b \quad a - b$

$\frac{\sin^2(y)}{1 - \cos(y)} \cdot \frac{1 + \cos(y)}{1 + \cos y}$

$\frac{\sin^2(y)(1 + \cos(y))}{1 - \cos^2(y)}$ Pythag. ident

$\frac{\sin^2(y)(1 + \cos(y))}{\sin^2(y)}$

$\boxed{1 + \cos(y)}$