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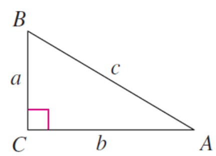
Applications Involving Right Triangles

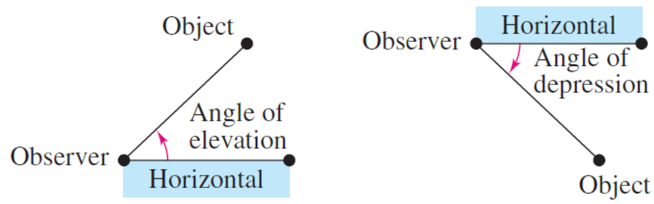
In this section, the three angles of a right triangle are denoted by the letters A, B, and C (where C is the right angle), and the lengths of the sides opposite these angles by the letters a, b, and c (where c is the hypotenuse).

Example 1

Given $B = 71°$ and $b = 14$, solve the triangle shown in the figure for all unknown sides and angles.



Recall from §4.3 that the term *angle of elevation* denotes the angle from the horizontal upward to an object and that the term *angle of depression* denotes the angle from the horizontal downward to an object. An angle of elevation and an angle of depression are shown in the figure below.



Example 2

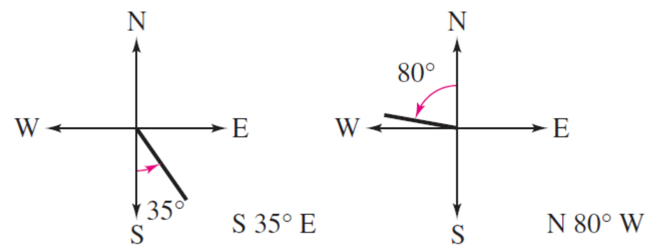
A ladder 20 feet long leans against the side of a house. The angle of elevation of the ladder is $80°$. Find the height from the top of the ladder to the ground.

Example 3

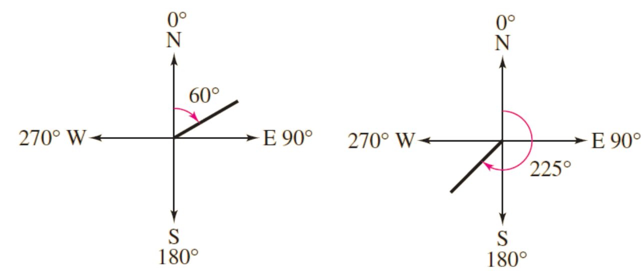
Find the angle of depression from the top of a lighthouse 250 feet above water level to the water line of a ship 2.5 miles offshore. (Hint: $5280 \text{ ft} = 1 \text{ mile}$.)

Trigonometry and Bearing

In surveying and navigation, directions are generally given in terms of **bearings**. A bearing measure the acute angle a path or line of sight makes with a fixed north-south line, as shown in the figure below. For instance, the bearing of S 35° E means 35 degrees east of south.



In *air navigation*, bearings are measured in degrees *clockwise* from north. Examples of air navigation bearings are shown below.



Example 4

A ship leaves port at noon and has a bearing of S 29° W. The ship sails at 20 knots (nautical miles per hour). How many nautical miles south and how many nautical miles west will the ship have traveled by 6:00 p.m.?

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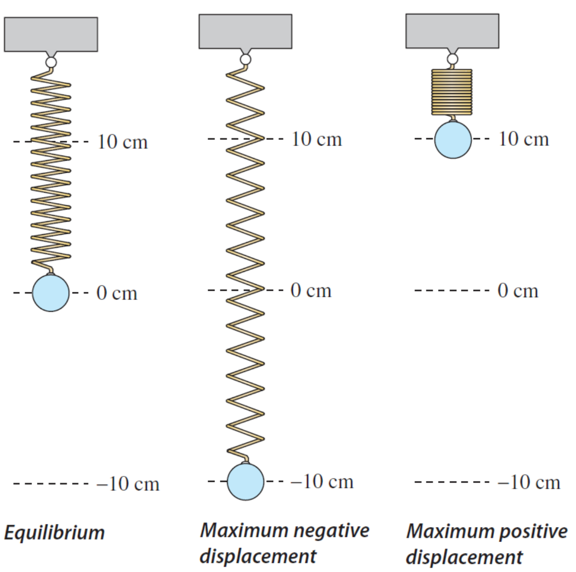
Harmonic Motion

The periodic nature of the trigonometric functions is useful for describing the motion of a point of an object that vibrates, oscillates, rotates, or is moved by wave motion.

For example, consider a ball that is bobbing up and down on the end of a spring, as shown in the figure. Suppose that 10 centimeters is the maximum distance the ball moves vertically upward or downward from its equilibrium (at-rest) position. Suppose further that the time it takes for the ball to move from its maximum displacement above zero to its maximum displacement below zero and back again is $t = 4$ seconds. Assuming the ideal conditions of perfect elasticity and no friction or air resistance, the ball would continue to move up and down in a uniform and regular manner.



From this spring you can conclude that the period (time for one complete cycle) of the motion is $\text{period} = 4$ seconds, its amplitude (maximum displacement from equilibrium) is $\text{amplitude} = 10$ centimeters, and its **frequency** (number of cycles per second) is $\text{frequency} = 1/4$ cycle per second.

Motion of this nature can be described by a sine or cosine function, and is called **simple harmonic motion**.

Definition of Simple Harmonic Motion

A point that moves on a coordinate line is said to be in **simple harmonic motion** if its distance d from the origin at time t is given by either

$$d = a\sin(\omega t) \quad \text{or} \quad d = a\cos(\omega t)$$

where a and ω are real numbers such that $\omega > 0$. The motion has amplitude $|a|$, period $2\pi/\omega$, and frequency $\omega/(2\pi)$.

Example 5

Find a model for single harmonic motion satisfying the specified conditions: no displacement at $t = 0$, amplitude 8 centimeters, period 2 seconds.

Example 6

For the simple harmonic motion described by the trigonometric function $d = \frac{1}{64}\sin(792\pi t)$, find (a) the maximum displacement, (b) the frequency, (c) the value of d when $t = 5$, and (d) the least positive value of t for which $d = 0$.

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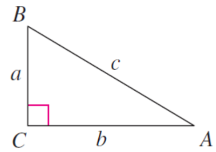
In Exercises 1-2, solve the triangle shown in the figure for all unknown sides and angles.



1. $A = 7.4°$, $a = 20.5$

2. $a = 25$, $c = 45$

3. A 100-ft line is attached to a kite. When the kite has pulled the line taut, the angle of elevation to the kite is approximately 50°. Approximate the height of the kite.

4. The sonar of a navy cruiser detects a submarine that is 4000 feet from the cruiser. The angle of depression to the submarine is 31.5°. How deep is the submarine below sea level?

5. An engineer erects a 75-foot vertical cellular-phone tower. Find the angle of elevation to the top of the tower from a point on level ground 95 feet from its base.

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6. An airplane flying at 600 miles per hour has a bearing of 52°. After flying for 1.5 hours, how far north and how far east has the plane traveled from its point of departure?

7. A ship is 45 miles east and 30 miles south of port. The captain wants to sail directly to port. What bearing should be taken?

8. Find a model for simple harmonic motion satisfying the specified conditions: displacement 3 inches at $t = 0$, amplitude 3 inches, period 1.5 seconds

9. For the simple harmonic motion described by the trigonometric function $d = \frac{1}{2}\cos(20\pi t)$, find (a) the maximum displacement, (b) the frequency, (c) the value of d when $t = 5$, and (d) the least positive value of t for which $d = 0$.