

Composition of Functions

For all x in the domains of f and f^{-1} , inverse functions have the properties

$$f(f^{-1}(x)) = x \quad \text{and} \quad f^{-1}(f(x)) = x.$$

Inverse Properties

- If $-1 \leq x \leq 1$ and $-\pi/2 \leq y \leq \pi/2$, then $\sin(\arcsin(x)) = x$ and $\arcsin(\sin(y)) = y$.
- If $-1 \leq x \leq 1$ and $0 \leq y \leq \pi$, then $\cos(\arccos(x)) = x$ and $\arccos(\cos(y)) = y$.
- If x is a real number and $-\pi/2 < y < \pi/2$, then $\tan(\arctan(x)) = x$ and $\arctan(\tan(y)) = y$.

Keep in mind that these inverse properties do not apply for arbitrary values of x and y . For instance,

$$\arcsin\left(\sin\left(\frac{3\pi}{2}\right)\right) = \arcsin(-1) = -\frac{\pi}{2} \neq \frac{3\pi}{2}.$$

In other words, the property $\arcsin(\sin(y)) = y$ is not valid for values of y outside the interval $-\pi/2 \leq y \leq \pi/2$.

Example 1

Use the properties of inverse functions to find the exact value of the expression.

(a) $\sin(\sin^{-1}(-0.3))$

(b) $\cos(\arccos(2.7))$

(c) $\tan^{-1}\left(\tan\left(\frac{11\pi}{6}\right)\right)$

(d) $\arcsin(\sin(3\pi))$

Example 2

Find the exact value of the expression $\cos(\arcsin(24/25))$.
(Hint: Make a sketch of a right triangle.)

Example 3

Write an algebraic expression that is equivalent to the expression $\sin(\arctan(x))$. (Hint: Make a sketch of a right triangle.)

In Exercises 1-12, use the properties of inverse functions to find the exact value of the expression.

1. $\tan(\arctan(35))$

2. $\sin(\arcsin(-1.1))$

3. $\arccos\left(\cos\left(\frac{2\pi}{3}\right)\right)$

4. $\sin^{-1}\left(\sin\left(\frac{7\pi}{4}\right)\right)$

5. $\cos^{-1}\left(\cos\left(\frac{3\pi}{2}\right)\right)$

6. $\tan^{-1}\left(\tan\left(-\frac{\pi}{3}\right)\right)$

7. $\arcsin\left(\sin\left(\frac{4\pi}{3}\right)\right)$

8. $\arccos\left(\cos\left(-\frac{7\pi}{6}\right)\right)$

9. $\arctan\left(\tan\left(\frac{5\pi}{4}\right)\right)$

10. $\cos(\arctan(-1))$

11. $\arcsin\left(\cos\left(-\frac{\pi}{6}\right)\right)$

12. $\arccos\left(\sin\left(-\frac{\pi}{6}\right)\right)$

In Exercises 13-15, find the exact value of the expression.
(Hint: Make a sketch of a right triangle.)

13. $\sin\left(\arctan\left(\frac{4}{3}\right)\right)$

14. $\csc\left(\arctan\left(-\frac{12}{5}\right)\right)$

15. $\sin\left(\arccos\left(-\frac{2}{3}\right)\right)$

In Exercises 16-18, write an algebraic expression that is equivalent to the expression. (Hint: Make a sketch of a right triangle.)

16. $\sin(\arccos(x + 2))$

17. $\tan\left(\arccos\left(\frac{x}{5}\right)\right)$

18. $\csc\left(\arctan\left(\frac{x}{\sqrt{7}}\right)\right)$