Trigonometry/Precalculus

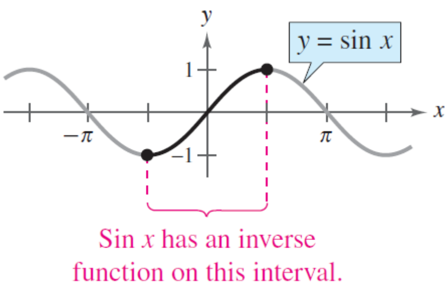
§4.7a Inverse Trigonometric Functions

Name ______________________________

Date __________________ Pd ______

Inverse Sine Function

For a function to have an inverse function, it must be one-to-one—that is, it must pass the Horizontal Line Test. In the figure below, it is obvious that $y = \sin(x)$ does not pass the test because different values of x yield the same y-value.



However, if you restrict the domain to the interval $-\pi/2 \le x \le \pi/2$ (corresponding to the black portion of the graph in the figure above), the following properties hold.

1. On the interval $-\pi/2 \le x \le \pi/2$, the function $y = \sin(x)$ is increasing.
2. On the interval $-\pi/2 \le x \le \pi/2$, $y = \sin(x)$ takes on its full range of values, $-1 \le \sin(x) \le 1$.
3. On the interval $-\pi/2 \le x \le \pi/2$, $y = \sin(x)$ is one-to-one.

So on the restricted domain $-\pi/2 \le x \le \pi/2$, $y = \sin(x)$ has a unique inverse function called the **inverse sine function**. It is denoted by

$$y = \arcsin(x) \quad \text{or} \quad y = \sin^{-1}(x).$$

The notation $\sin^{-1}(x)$ is consistent with the inverse function notation $f^{-1}(x)$. The $\arcsin(x)$ notation (read "the arcsine of x") comes from the association of a central angle with its intercepted *arc length* on a unit circle. So $\arcsin(x)$ means the angle (or arc) whose sine is x. Both notations, $\arcsin(x)$ and $\sin^{-1}(x)$, are commonly used in mathematics, so remember that $\sin^{-1}(x)$ denotes the *inverse* sine function rather than $1/\sin(x)$. The values of $\arcsin(x)$ lie in the interval $-\pi/2 \le \arcsin(x) \le \pi/2$.

Definition of Inverse Sine Function

The **inverse sine function** is defined by

$$y = \arcsin(x) \quad \text{if and only if} \quad \sin(y) = x$$

where the domain is $-1 \le x \le 1$ and the range is $-\pi/2 \le y \le \pi/2$.

When evaluating the inverse sine function, it helps to remember the phrase "the arcsine of x is the angle (or number) whose sine is x."

Example 1

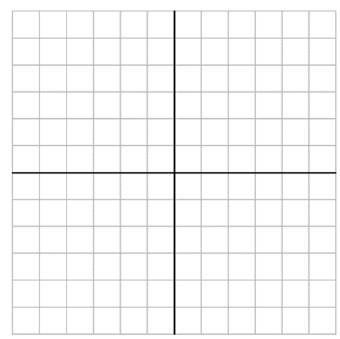
Find the exact value of each expression without using a calculator.

(a) $\arcsin\left(\frac{1}{2}\right)$

(b) $\arcsin(0)$

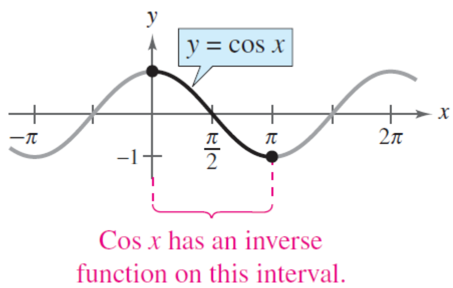
Example 2

Sketch the graph of the function $y = \arcsin(x)$ by hand.



Other Inverse Trigonometric Functions

The cosine function is decreasing and one-to-one on the interval $0 \le x \le \pi$, as shown in the figure below.



Consequently, on this interval the cosine function has an inverse function—the **inverse cosine function**—denoted by

$$y = \arccos(x) \quad \text{or} \quad y = \cos^{-1}(x).$$

Because $y = \arccos(x)$ and $x = \cos(y)$ are equivalent for $0 \le y \le \pi$, their graphs are the same, and can be confirmed by the following table of values.

y	0	$\pi/3$	$\pi/2$	$2\pi/3$	π
$x = \cos(y)$	1	$1/2$	0	$-1/2$	-1

Similarly, you can define an **inverse tangent function** by restricting the domain of $y = \tan(x)$ to the interval $-\pi/2 < x < \pi/2$. The following list summarizes the definitions of the six inverse trigonometric functions.

Definitions of the Inverse Trigonometric Functions

- $y = \arcsin(x)$ if and only if $\sin(y) = x$
  Domain: $-1 \le x \le 1$
  Range: $-\pi/2 \le y \le \pi/2$
- $y = \arccos(x)$ if and only if $\cos(y) = x$
  Domain: $-1 \le x \le 1$
  Range: $0 \le y \le \pi$
- $y = \arctan(x)$ if and only if $\tan(y) = x$
  Domain: $-\infty < x < \infty$
  Range: $-\pi/2 < y < \pi/2$
- $y = \text{arccot}(x)$ if and only if $\cot(y) = x$
  Domain: $-\infty < x < \infty$
  Range: $0 < y < \pi$
- $y = \text{arcsec}(x)$ if and only if $\sec(y) = x$
  Domain: $x \le -1$ or $x \ge 1$
  Range: $0 \le y < \pi/2$ or $\pi/2 < y \le \pi$
- $y = \text{arccsc}(x)$ if and only if $\csc(y) = x$
  Domain: $x \le -1$ or $x \ge 1$
  Range: $-\pi/2 \le y < 0$ or $0 < y \le \pi/2$

Example 3

Find the exact value of each expression without using a calculator.

(a) $\arccos\left(\dfrac{1}{2}\right)$

(b) $\arccos(0)$

(c) $\arctan\left(\dfrac{\sqrt{3}}{3}\right)$

(d) $\arctan(-1)$

Example 4

Use a calculator to approximate the value of $\cos^{-1}(0.75)$. Leave your answer accurate to four decimal places.

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In Exercises 1-9, find the exact value of each expression without using a calculator.

1. arcsin(1)

2. arccos(1)

3. arctan(1)

4. $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

5. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

6. $\arctan(-\sqrt{3})$

7. $\arccos\left(-\frac{1}{2}\right)$

8. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

9. $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

In Exercises 10-12, use a calculator to approximate the value of the expression. Leave your answer accurate to four decimal places.

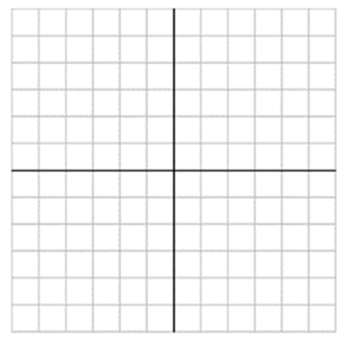
10. $\sin^{-1}(0.56)$

11. $\arccos(-0.7)$

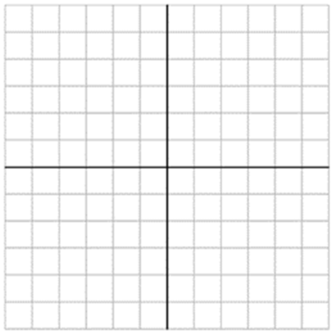
12. $\tan^{-1}(5.9)$

In Exercises 13-17, sketch the graph of the function by hand.

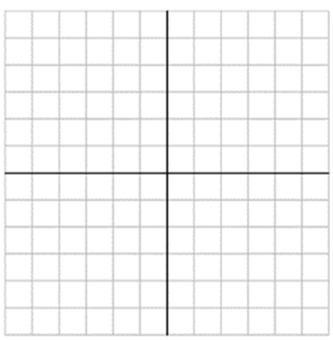
13. $y = \arccos(x)$



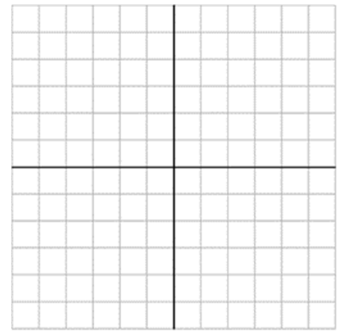
14. $y = \arctan(x)$



15. $y = \text{arccot}(x)$



16. $y = \text{arcsec}(x)$



17. $y = \text{arccsc}(x)$