Name ____________________

Date ______________ Pd ______

Graph of the Reciprocal Functions

The graphs of the two remaining trigonometric functions can be obtained from the graphs of the sine and cosine functions using the reciprocal identities

$$\csc(x) = \frac{1}{\sin(x)} \quad \text{and} \quad \sec(x) = \frac{1}{\cos(x)}.$$

For instance, at a given value of x, the y-coordinate for $\sec(x)$ is the reciprocal of the y-coordinate for $\cos(x)$. Of course, when $\cos(x) = 0$, the reciprocal does not exist. Near such values of x, the behavior of the secant function is similar to that of the tangent function. In other words, the graphs of

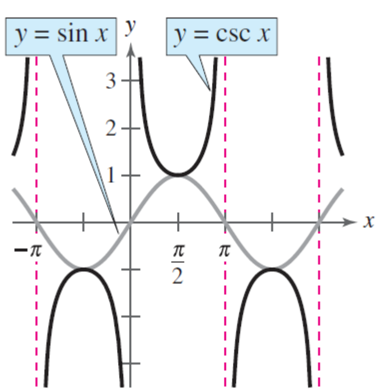
$$\tan(x) = \frac{\sin(x)}{\cos(x)} \quad \text{and} \quad \sec(x) = \frac{1}{\cos(x)}$$

have vertical asymptotes at $x = \pi/2 + n\pi$, where n is an integer (i.e., the values at which the cosine is zero). Similarly,

$$\cot(x) = \frac{\cos(x)}{\sin(x)} \quad \text{and} \quad \csc(x) = \frac{1}{\sin(x)}$$

have vertical asymptotes where $\sin(x) = 0$—that is, at $x = n\pi$.

To sketch the graph of a secant or cosecant function, you should first make a sketch of its reciprocal function. For instance, to sketch the graph of $y = \csc(x)$, first sketch the graph of $y = \sin(x)$. Then take the reciprocals of the y-coordinates to obtain points on the graph of $y = \csc(x)$.



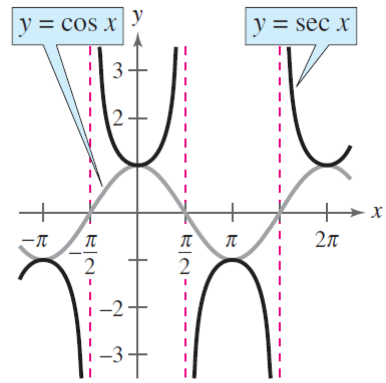
Period: 2π

Domain: all real numbers x, except $x = n\pi$

Range: $y \le -1$ or $y \ge 1$

Vertical asymptotes: $x = n\pi$

Symmetry: origin



Period: 2π

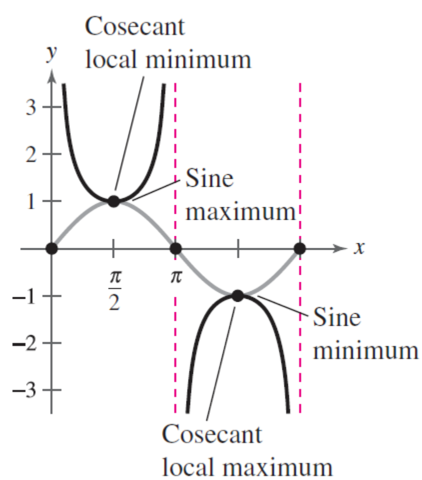
Domain: all real numbers x, except $x = \frac{\pi}{2} + n\pi$

Range: $y \le -1$ or $y \ge 1$

Vertical asymptotes: $x = \frac{\pi}{2} + n\pi$

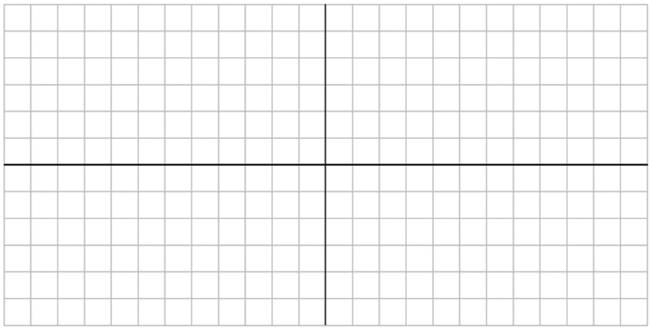
Symmetry: y-axis

In comparing the graphs of the cosecant and secant functions with those of the sine and cosine functions, note that the "hills" and "valleys" are interchanged. For example, a hill (or maximum point) on the sine curve corresponds to a valley (a local minimum) on the cosecant curve, and a valley (or minimum point) on the sine curve corresponds to a hill (a local maximum) on the cosecant curve. Additionally, x-intercepts of the sine and cosine functions become vertical asymptotes of the cosecant and secant functions, respectively.



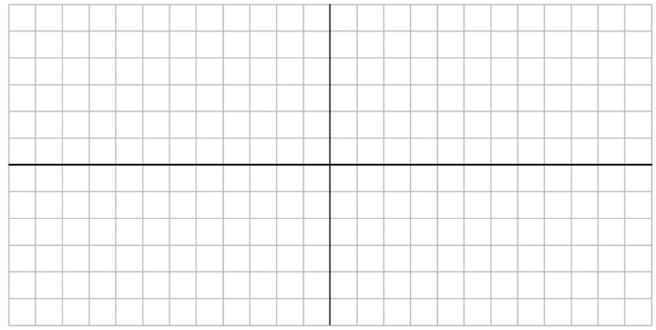
Example 1

Sketch the graph of the function $y = 1.5\csc(3x)$ by hand. (Include a maximum of two full periods.) Identify the period and phase shift of the graph.



Example 2

Sketch the graph of the function $y = -0.75\sec(4x - \pi)$ by hand. (Include a maximum of two full periods.) Identify the period and phase shift of the graph.

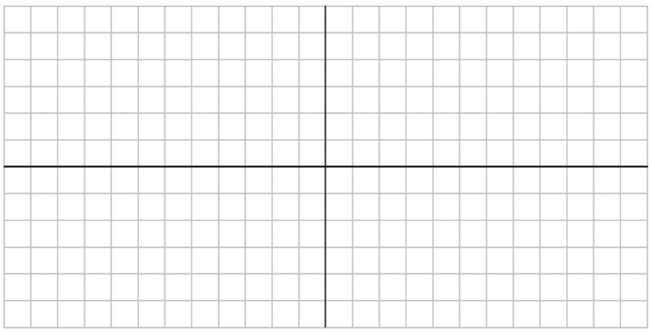


Trigonometry/Precalculus | Name ______________________

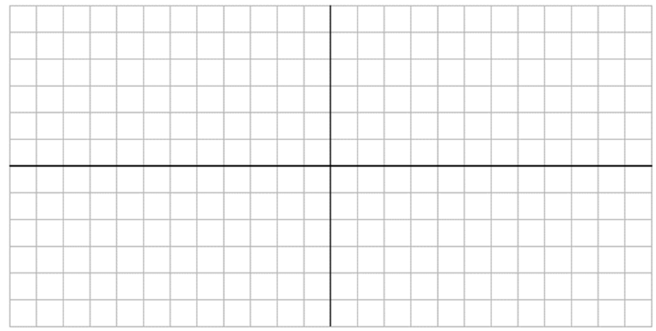
§4.6b Graphs of Secant and Cosecant Functions HOMEWORK | Date ______________ Pd ______

In Exercises 1-4, sketch the graph of the function by hand. (Include a maximum of two full periods.) Identify the period and phase shift of the graph.

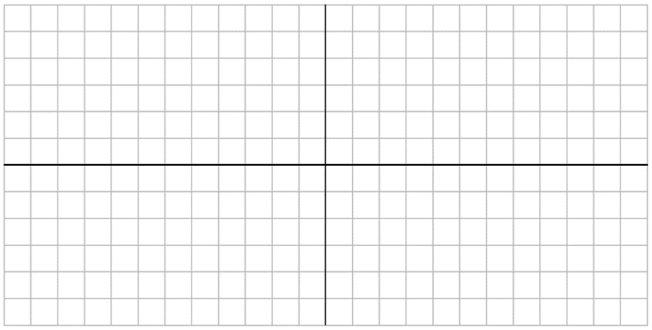
1. $y = 0.5\csc(2x)$



2. $y = 1.5\sec\left(\frac{2x}{3}\right)$



3. $y = -\csc\left(\frac{x}{2} - \frac{\pi}{2}\right)$



4. $y = -0.5\sec\left(2x - \frac{\pi}{2}\right)$