Trigonometry/Precalculus | Name ____________________

§4.6a Graphs of Tangent and Cotangent Functions | Date ____________ Pd ______

## Graph of the Tangent Function

The graph of $y = \tan(x)$ is symmetric with respect to the origin. You also know from the identity $\tan(x) = \sin(x)/\cos(x)$ that the tangent function is undefined at values at which $\cos(x) = 0$. Two such values are $x = \pm\pi/2 \approx \pm 1.5708$.

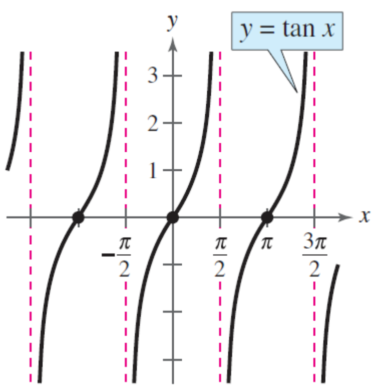
| $x$ | $-\frac{\pi}{2}$ | $-1.57$ | $-1.5$ | $-\frac{\pi}{4}$ | 0 | $\frac{\pi}{4}$ | 1.5 | 1.57 | $\frac{\pi}{2}$ |
|---|---|---|---|---|---|---|---|---|---|
| $\tan x$ | Undef. | $-1255.8$ | $-14.1$ | $-1$ | 0 | 1 | 14.1 | 1255.8 | Undef. |

← $\tan x$ approaches $-\infty$ as $x$ approaches $-\pi/2$ from the right.

$\tan x$ approaches $\infty$ as $x$ approaches $\pi/2$ from the left. →

As indicated in the table, $\tan(x)$ increases without bound as $x$ approaches $\pi/2$ from the left, and it decreases without bound as $x$ approaches $-\pi/2$ from the right. So, the graph of $y = \tan(x)$ has *vertical asymptotes* at $x = \pi/2$ and $x = -\pi/2$, as shown in the figure below.

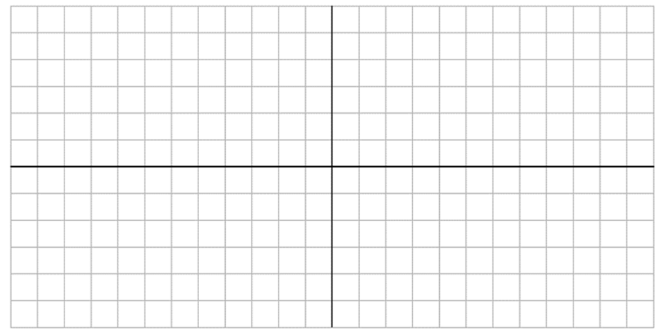
Moreover, because the period of the tangent function is $\pi$, vertical asymptotes also occur at $x = \pi/2 + n\pi$, where $n$ is an integer. The domain of the tangent function is the set of all real numbers other than $x = \pi/2 + n\pi$, and the range is the set of all real numbers.




Sketching the graph of $y = a\tan\left(b(x-h)\right)$ is similar to sketching the graph of $y = a\sin\left(b(x-h)\right)$ in that you locate key points that identify the intercepts and asymptotes. Two consecutive asymptotes can be found by solving the equations $b(x-h) = -\pi/2$ and $b(x-h) = \pi/2$. The midpoint between two consecutive asymptotes is an $x$-intercept of the graph. The period of the function $y = a\tan\left(b(x-h)\right)$ is the distance between two consecutive asymptotes. The amplitude of a tangent function is not defined. After plotting the asymptotes and the $x$-intercept, plot a few additional points between the two asymptotes and sketch one cycle. Finally, sketch one or two additional cycles to the left and right.

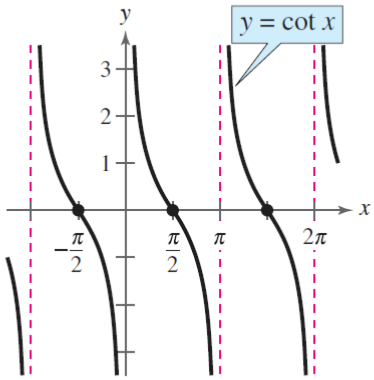
## Example 1

Sketch the graph of the functions $y = 0.5\tan(x)$ and $y = -2\tan(2x+\pi)$ by hand. (Include a maximum of two full periods.) Identify the period and phase shift of the graph.



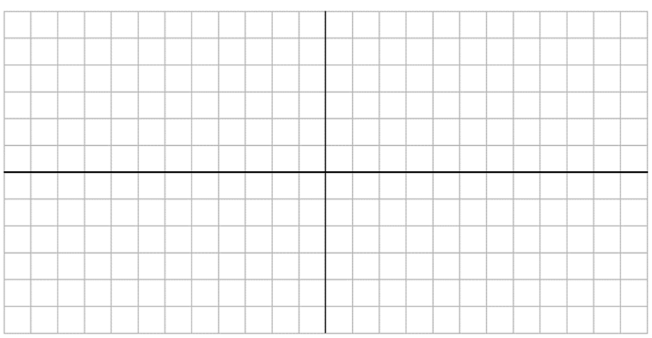
## Graph of the Cotangent Function

The graph of the cotangent function is similar to the graph of the tangent function. It also has a period of $\pi$. However, from the identity $\cot(x) = \cos(x)/\sin(x)$, you can see that the cotangent function has vertical asymptotes when $\sin(x)$ is zero, which occurs at $x = n\pi$, where $n$ is an integer.




## Example 2

Sketch the graph of the function $y = \frac{1}{2}\cot\left(\frac{x}{2}\right)$ by hand. Identify the period and phase shift of the graph.



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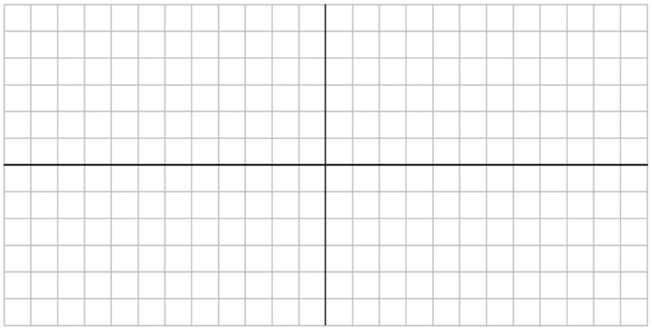
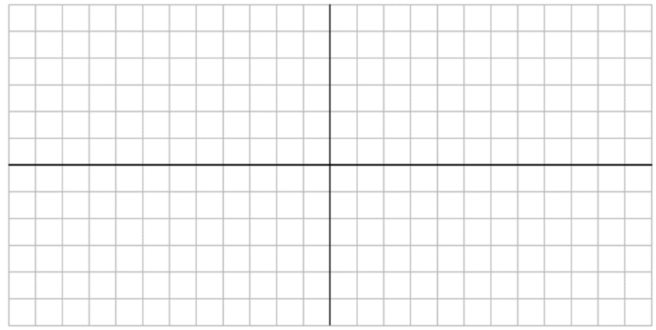
§4.6a Graphs of Tangent and Cotangent Functions HOMEWORK | Date ______________ Pd ______

In Exercises 1-4, sketch the graph of the function by hand. (Include a maximum of two full periods.) Identify the period and phase shift of the graph.

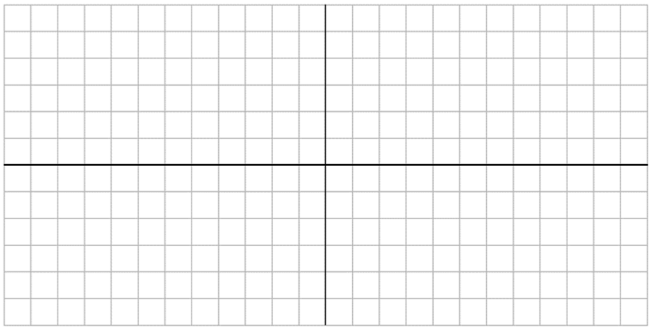
1. $y = 2\tan(3x)$



2. $y = -\frac{5}{2}\tan\left(\frac{2x}{3} + \frac{3\pi}{8}\right)$



3. $y = -1.5\cot\left(\frac{3x}{4}\right)$



4. $y = 2\cot\left(\frac{3x}{2} - \frac{\pi}{3}\right)$