Trigonometry/Precalculus Name _____

§4.6a Graphs of Tangent and Cotangent Functions Date _____ Pd _____

TANGENT GRAPH

Sketch the graph of $y = \tan\left(\frac{x}{2} + \frac{\pi}{12}\right)$ by hand. Identify the amplitude, period, and phase shift of the graph.

Characteristics

- To determine amplitude: Find $|a|$
- To determine period: Determine π/b.
- To determine phase shift:
  - Set the argument (part of the trigonometric function inside parentheses) equal to zero.
  - Solve the resulting equation.
  - If the resulting value is positive, the PS is to the right.
  - If the resulting value is negative, the PS is to the left.

- Amplitude 1
- Period 2π
- Phase shift $\pi/6$ left

Characteristic Calculations

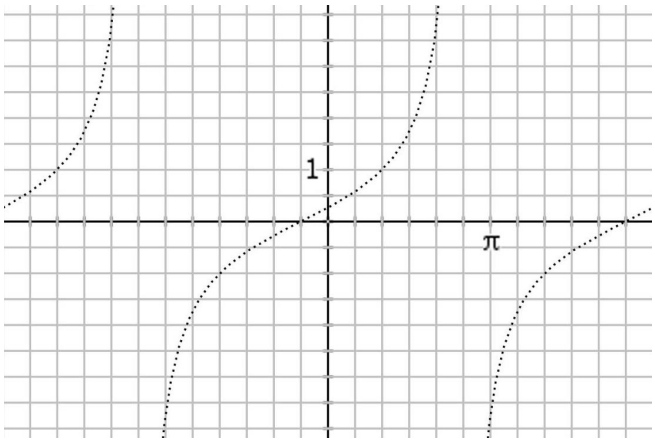
- Period: $\frac{\pi}{1/2} = \frac{\pi}{1/2} \cdot \frac{2}{2} = \frac{2\pi}{1} = 2\pi$
- Phase shift: $\frac{x}{2} + \frac{\pi}{12} = 0 \Rightarrow \frac{x}{2} = -\frac{\pi}{12} \Rightarrow x = -\frac{2\pi}{12} = -\frac{\pi}{6}$

Determining Key Points

old x ∠s	$-\pi/2$	$-\pi/4$	0	$\pi/4$	$\pi/2$
old x □s	–3	–1.5	0	1.5	3
old y	VA	–1	0	1	VA

- To determine new x-values:
  - Set the argument equal to each old x.
  - Solve the resulting equation.
  - The resulting values tell the location of the new x.
- To determine new y-values:
  - Replace the tangent term with each old y.
  - Evaluate the resulting expression.
  - The values give the new y-values.

- Suggestion:
  - Start with VAs—recommended to utilize old $x = \pi/2$.
  - VAs are separated horizontally by distance equal to period.
  - Determine middle point (old $y = 0$).
  - Determine right point (old $y = 1$).
  - Left point is equidistant horizontally from middle point, though you may want to calculate the y-value.



Key Point Calculations

- Vertical asymptotes
  - $\frac{x}{2} + \frac{\pi}{12} = \frac{\pi}{2} \Rightarrow \frac{x}{2} = \frac{6\pi}{12} - \frac{\pi}{12} = \frac{5\pi}{12} \Rightarrow x = \frac{5\pi}{6}$

    OR $\frac{x}{2} + \frac{1}{2} = 3 \Rightarrow x + 1 = 6 \Rightarrow x = 5$ squares
  - Distance between VAs: 2π or 12 squares
- Middle point
  - $\frac{x}{2} + \frac{\pi}{12} = 0 \Rightarrow \frac{x}{2} = -\frac{\pi}{12} \Rightarrow x = -\frac{\pi}{6}$

    OR $\frac{x}{2} + \frac{1}{2} = 0 \Rightarrow x + 1 = 0 \Rightarrow x = -1$ square
  - $y = 0$
- Right point
  - $\frac{x}{2} + \frac{\pi}{12} = \frac{\pi}{4} \Rightarrow \frac{x}{2} = \frac{3\pi}{12} - \frac{\pi}{12} = \frac{\pi}{6} \Rightarrow x = \frac{\pi}{3}$

    OR $\frac{x}{2} + \frac{1}{2} = \frac{3}{2} \Rightarrow x + 1 = 3 \Rightarrow x = 2$ squares
  - $y = 1$
- Left point
  - $y = -1$

COTANGENT GRAPH

Sketch the graph of $y = 2\cot\left(x - \frac{\pi}{3}\right)$ by hand. Identify the amplitude, period, and phase shift of the graph.

Characteristics (same as TANGENT graph)

- To determine amplitude: Find $|a|$
- To determine period: Determine π/b.
- To determine phase shift:
  - Set the argument (part of the trigonometric function inside parentheses) equal to zero. Solve the resulting equation.
  - If the resulting value is positive, the PS is to the right.
  - If the resulting value is negative, the PS is to the left.

- Amplitude 2
- Period π
- Phase shift $\pi/3$ right

Characteristic Calculations

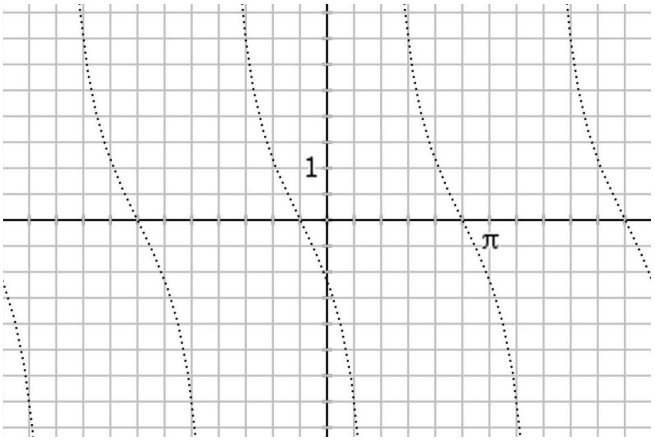
- Period: $\frac{\pi}{1} = \pi$
- Phase shift: $x - \frac{\pi}{3} = 0 \Rightarrow x = \frac{\pi}{3}$

Determining Key Points

old x ∠s	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
old x □s	0	1.5	3	4.5	6
old y	VA	1	0	–1	VA

- To determine new x-values:
  - Set the argument equal to each old x.
  - Solve the resulting equation.
  - The resulting values tell the location of the new x.
- To determine new y-values:
  - Replace the cotangent term with each old y.
  - Evaluate the resulting expression.
  - The values give the new y-values.

- Suggestion:
  - Start with VAs—recommended to utilize old $x = 0$.
  - VAs are separated horizontally by distance equal to period.
  - Determine middle point (old $y = 0$).
  - Determine left point (old $y = 1$).
  - Right point is equidistant horizontally from middle point, though you may want to calculate the y-value.



Key Point Calculations

- Vertical asymptotes
  - $x - \frac{\pi}{3} = 0 \Rightarrow x = \frac{\pi}{3}$ OR
    $x - 2 = 0 \Rightarrow x = 2$ squares
  - Distance between VAs: π or 6 squares
- Middle point
  - $x - \frac{\pi}{3} = \frac{\pi}{2} \Rightarrow x = \frac{3\pi}{6} + \frac{2\pi}{6} = \frac{5\pi}{6}$ OR
    $x - 2 = 3 \Rightarrow x = 5$ squares
  - $y = 2(0) = 0$
- Left point
  - $x - \frac{\pi}{3} = \frac{\pi}{4} \Rightarrow x = \frac{3\pi}{12} + \frac{4\pi}{12} = \frac{7\pi}{12}$ OR
    $x - 2 = 1.5 \Rightarrow x = 3.5$ squares
  - $y = 2(1) = 2$
- Right point
  - $y = 2(-1) = -2$

Trigonometry/Precalculus Name _____

§4.6b Graphs of Secant and Cosecant Functions Date _____ Pd _____

SECANT GRAPH

Sketch the graph of $y = \sec\left(2x - \frac{\pi}{3}\right)$ by hand. Identify the period and phase shift of the graph.

Characteristics (same as SINE/COSINE)

- To determine period: Determine $2\pi/b$.
- To determine phase shift:
  - Set the argument (part of the trigonometric function inside parentheses) equal to zero.
  - Solve the resulting equation.
  - If the resulting value is positive, the PS is to the right.
  - If the resulting value is negative, the PS is to the left.
- Period π
- Phase shift $\pi/6$ right

Characteristic Calculations

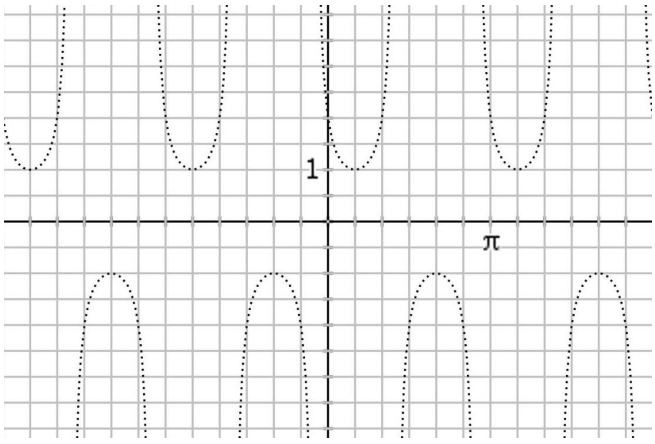
- Period: $\frac{2\pi}{2} = \pi$
- Phase shift: $2x - \frac{\pi}{3} = 0 \Rightarrow 2x = \frac{\pi}{3} \Rightarrow x = \frac{\pi}{6}$

Determining Key Points

old x ∠s	0	$\pi/3$	$\pi/2$	$2\pi/3$	π	$4\pi/3$	$3\pi/2$	$5\pi/3$
old x □s	0	2	3	4	6	8	9	10
old y	1	2	VA	–2	–1	–2	VA	2

Note: This chart is exactly like the $\cos(x)$ chart, except the y-values are reciprocals.

- To determine new x-values:
  - Set the argument equal to each old x.
  - Solve the resulting equation.
  - The resulting values tell the location of the new x.
- To determine new y-values:
  - Replace the secant term with each old y.
  - Evaluate the resulting expression.
  - The values give the new y-values.
- Suggestion:
  - Start with VAs—recommended to utilize old $x = \pi/2$.
  - Determine middle point (old $y = 1$ or $y = -1$).
  - Determine left or right point (old $y = 2$ or $y = -2$).
  - Other point is equidistant horizontally from middle point.
  - VAs and other points are separated horizontally by distance equal to half the period, though you may want to calculate the y-values.



Key Point Calculations

- Vertical asymptotes
  - $2x - \frac{\pi}{3} = \frac{\pi}{2} \Rightarrow 2x = \frac{3\pi}{6} + \frac{2\pi}{6} = \frac{5\pi}{6} \Rightarrow x = \frac{5\pi}{12}$ OR
    $2x - 2 = 3 \Rightarrow 2x = 5 \Rightarrow x = 2.5$ squares
- First middle point
  - $2x - \frac{\pi}{3} = 0 \Rightarrow 2x = \frac{\pi}{3} \Rightarrow x = \frac{\pi}{6}$ OR
    $2x - 2 = 0 \Rightarrow 2x = 2 \Rightarrow x = 1$ square
  - $y = 1$
- First right point
  - $2x - \frac{\pi}{3} = \frac{\pi}{3} \Rightarrow 2x = \frac{2\pi}{3} \Rightarrow x = \frac{\pi}{3}$ OR
    $2x - 2 = 2 \Rightarrow 2x = 4 \Rightarrow x = 2$ squares
  - $y = 2$
- Second middle point
  - $y = -1$
- Second right point
  - $y = -2$
- Distance between VAs and points: $\pi/2$ or 3 squares

COSECANT GRAPH

Sketch the graph of $y = -0.5\csc\left(x + \frac{2\pi}{3}\right)$ by hand. Identify the period and phase shift of the graph.

Characteristics (same as SINE/COSINE/SECANT graph)

- To determine period: Determine $2\pi/b$.
- To determine phase shift:
  - Set the argument (part of the trigonometric function inside parentheses) equal to zero.
  - Solve the resulting equation.
  - If the resulting value is positive, the PS is to the right.
  - If the resulting value is negative, the PS is to the left.
- Period 2π
- Phase shift $2\pi/3$ left

Characteristic Calculations

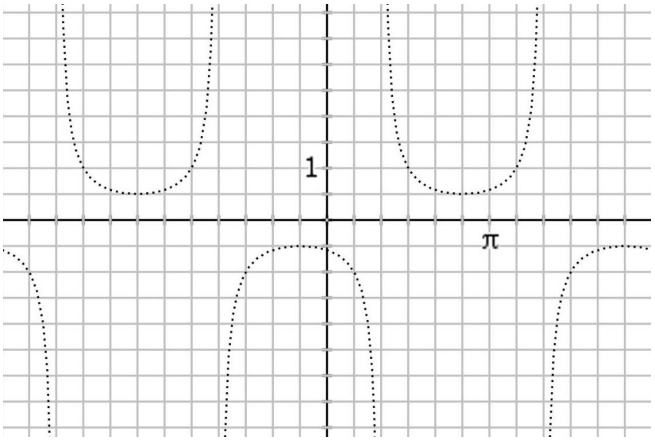
- Period: $\frac{2\pi}{1} = 2\pi$
- Phase shift: $x + \frac{2\pi}{3} = 0 \Rightarrow x = -\frac{2\pi}{3}$

Determining Key Points

old x ∠s	0	$\pi/6$	$\pi/2$	$5\pi/6$	π	$7\pi/6$	$3\pi/2$	$11\pi/6$
old x □s	0	1	3	5	6	7	9	11
old y	VA	2	1	2	VA	–2	–1	–2

Note: This chart is exactly like the $\sin(x)$ chart, except the y-values are reciprocals.

- To determine new x-values:
  - Set the argument equal to each old x.
  - Solve the resulting equation.
  - The resulting values tell the location of the new x.
- To determine new y-values:
  - Replace the cosecant term with each old y.
  - Evaluate the resulting expression.
  - The values give the new y-values.
- Suggestion:
  - Start with VAs—recommended to utilize old $x = 0$.
  - Determine middle point (old $y = 1$ or $y = -1$).
  - Determine left or right point (old $y = 2$ or $y = -2$).
  - Other point is equidistant horizontally from middle point.
  - VAs and other points are separated horizontally by distance equal to half the period, though you may want to calculate the y-values.



Key Point Calculations

- Vertical asymptotes
  - $x + \frac{2\pi}{3} = 0 \Rightarrow x = -\frac{2\pi}{3}$ OR
    $x + 4 = 0 \Rightarrow x = -4$ squares
- First middle point
  - $x + \frac{2\pi}{3} = \frac{\pi}{2} \Rightarrow x = \frac{3\pi}{6} - \frac{4\pi}{6} = -\frac{\pi}{6}$ OR
    $x + 4 = 3 \Rightarrow x = -1$ square
  - $y = -0.5(1) = -0.5$
- First left point
  - $x + \frac{2\pi}{3} = \frac{\pi}{6} \Rightarrow x = \frac{\pi}{6} - \frac{4\pi}{6} = -\frac{3\pi}{6} = -\frac{\pi}{2}$ OR
    $x + 4 = 1 \Rightarrow x = -3$ squares
  - $y = -0.5(2) = -1$
- Second middle point
  - $y = -0.5(-1) = 0.5$
- Second left point
  - $y = -0.5(-2) = 1$
- Distance between VAs and points: π or 6 squares