Trigonometry/Precalculus — Name ______________________

§4.5b Graphs of Sine and Cosine Functions — Date ______________ Pd ______

Translations of Sine and Cosine Curves

The constant h in the general equations

$$y = a\sin\left(b(x-h)\right) \text{ and } y = a\cos\left(b(x-h)\right)$$

creates *horizontal translations* (shifts) of the basic sine and cosine curves. Comparing $y = a\sin(bx)$ with $y = a\sin\left(b(x-h)\right)$, you find that the graph of $y = a\sin\left(b(x-h)\right)$ completes one cycle from $b(x-h)=0$ to $b(x-h)=2\pi$. By solving for x, you can find the interval for one cycle to be

$$\overbrace{h}^{\text{Left endpoint}} \le x \le \overbrace{h + \underbrace{\frac{2\pi}{b}}_{\text{Period}}}^{\text{Right endpoint}}.$$

This implies that the period of $y = a\sin\left(b(x-h)\right)$ is $2\pi/b$, and the graph of $y = a\sin(bx)$ is shifted by an amount h. The number h is the **phase shift**.

Graphs of Sine and Cosine Functions

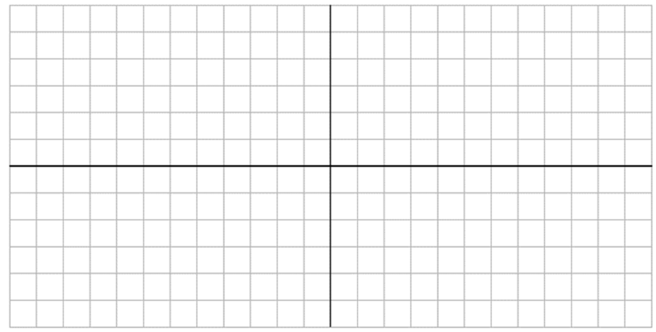
The graphs of $y = a\sin\left(b(x-h)\right)$ and $y = a\cos\left(b(x-h)\right)$ have the following characteristics. (Assume $b > 0$.)

$$\text{amplitude} = |a| \qquad \text{period} = 2\pi/b$$

The left and right endpoints of a one-cycle interval can be determined by solving the equations $b(x-h)=0$ and $b(x-h)=2\pi$.

Example 1

Sketch the graph of the function $y = -3\cos(6x+\pi)$ by hand. (Include a maximum of two full periods.) Identify the amplitude, period, and phase shift of the graph.



The final type of transformation is the *vertical translation* caused by the constant k in the equations

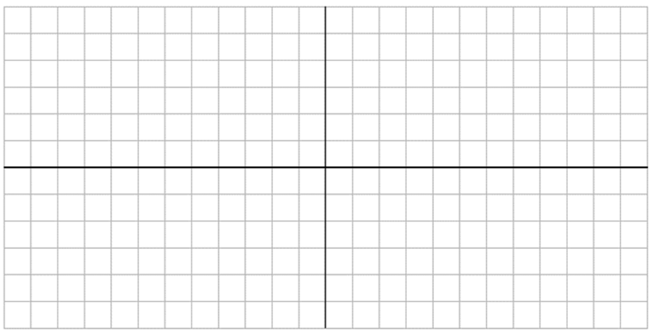
$$y = a\sin\big(b(x-h)\big)+k \text{ and } y = a\cos\big(b(x-h)\big)+k.$$

The shift is k units upward for $k>0$ and k units downward for $k<0$. In other words, the graph oscillates about the horizontal line $y=k$ instead of about the *x*-axis.

Example 2

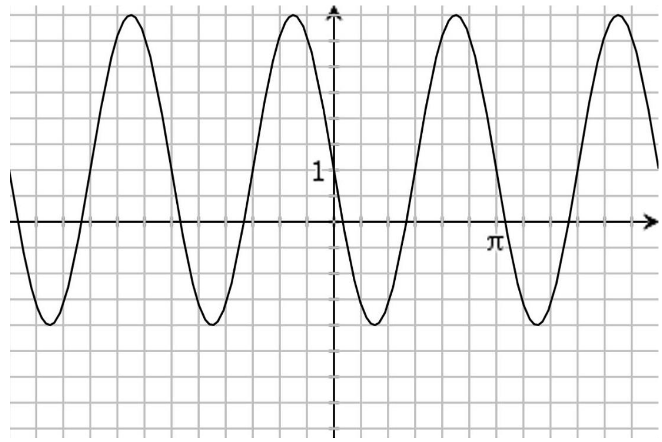
Sketch the graph of the function $y = 5\cos\left(\frac{2x}{3}\right)-4$ by hand.

Identify the amplitude, period, phase shift, and vertical shift of the graph.



Example 3

Find a, b, h, and k for the function $f(x) = a\sin\big(b(x-h)\big)+k$ such that the graph of f matches the graph shown.

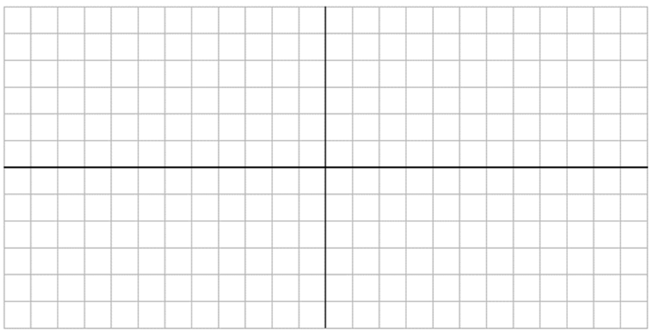


Trigonometry/Precalculus | Name ______________________

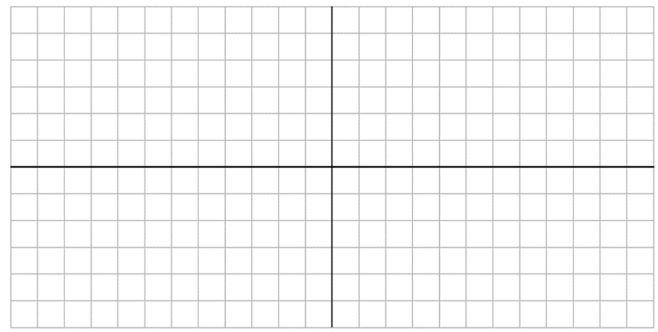
§4.5b Graphs of Sine and Cosine Functions HOMEWORK | Date ____________ Pd ______

In Exercises 1-3, sketch the graph of the function by hand. (Include a maximum of two full periods.) Identify the amplitude, period, phase shift, and vertical shift of the graph.

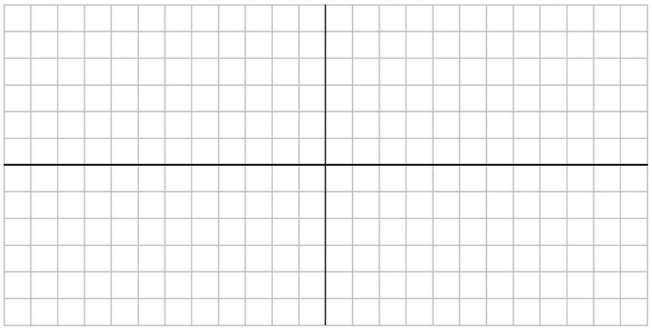
1. $y = \sin\left(x - \frac{\pi}{6}\right) + 2$



2. $y = \frac{3}{2}\cos\left(4x - \frac{2\pi}{3}\right)$

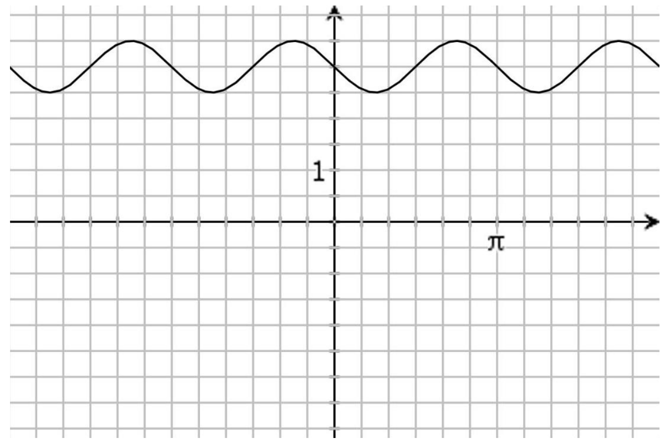


3. $y = -\frac{1}{2}\sin\left(2x + \frac{4\pi}{3}\right) + 1$



In Exercises 4-5, find a, b, h, and k for the function $f(x) = a\sin(b(x-h)) + k$ such that the graph of f matches the graph shown.

4.



5.