Trigonometry/Precalculus

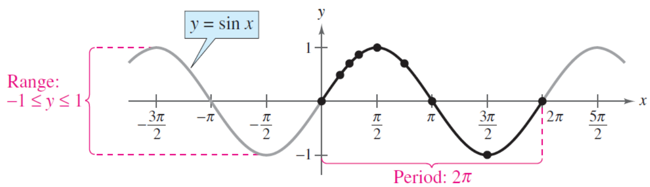
§4.5a Graphs of Sine and Cosine Functions

Name ____________________

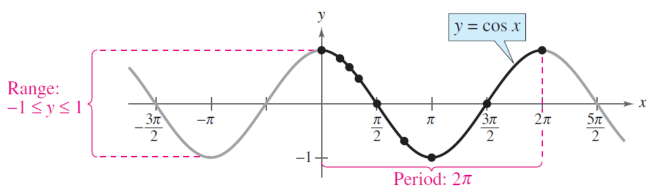
Date ______________ Pd ______

Basic Sine and Cosine Curves

The graph of the sine function is a **sine curve**. In the figure below, the black portion of the graph represents one period of the function and is called **one cycle** of the sine curve. The gray portion of the graph indicates that the basic sine wave repeats indefinitely to the right and left.



The graph of the cosine function is shown below.



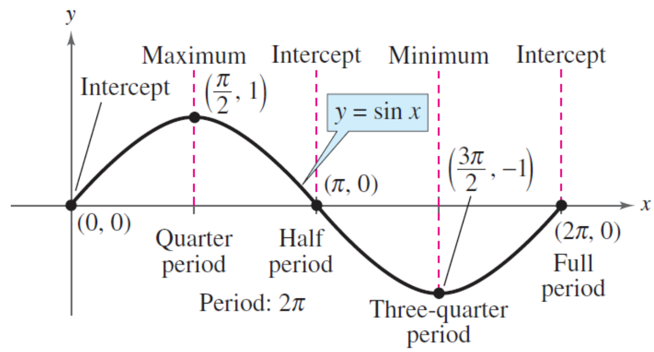
Recall from §4.2 that the domain of the sine and cosine functions is the set of all real numbers. Moreover, the range of each function is $-1 \le y \le 1$, and each function has a period of 2π.

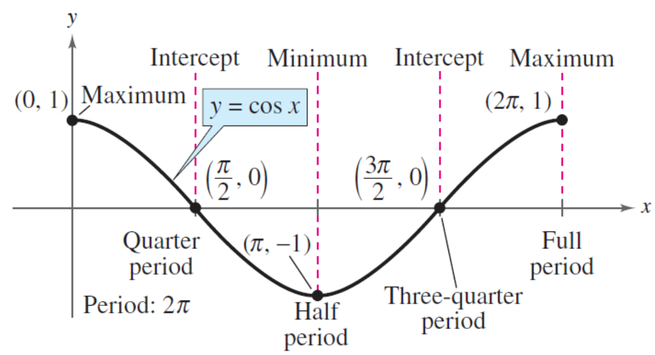
The table below lists key points on the graphs of $y = \sin(x)$ and $y = \cos(x)$.

x	0	$\pi/6$	$\pi/3$	$\pi/2$	π	$3\pi/2$	2π
$\sin(x)$	0	1/2		1	0	−1	0
$\cos(x)$	1		1/2	0	−1	0	1

Note in the figures above that the sine curve is symmetric with respect to the *origin*, whereas the cosine curve is symmetric with respect to the *y-axis*.

To sketch the graphs of the basic sine and cosine functions by hand, it helps to note nine key points in one period of each graph: the three *intercepts*, the maximum *point*, the *minimum point*, and the four *halfway points* (which are not shown in the figures below).





Amplitude and Period of Sine and Cosine Curves

In the rest of this section, you will study the graphic effect of each of the constants a, b, h, and k in equations of the forms

$$y = a\sin\big(b(x-h)\big)+k \qquad \text{and} \qquad y = a\cos\big(b(x-h)\big)+k.$$

The constant factor a in $y = a\sin(x)$ acts as a *scaling factor*—a *vertical stretch* or *vertical shrink* of the basic sine curve. If $|a| > 1$, the basic sine curve is stretched, and if $|a| < 1$, the basic sine curve is shrunk.

The result is that the graph of $y = a\sin(x)$ ranges between $-a$ and a instead of between -1 and 1. The absolute value of a is the **amplitude** of the function $y = a\sin(x)$. The range of the function $y = a\sin(x)$ for $a > 0$ is $-a \le y \le a$.

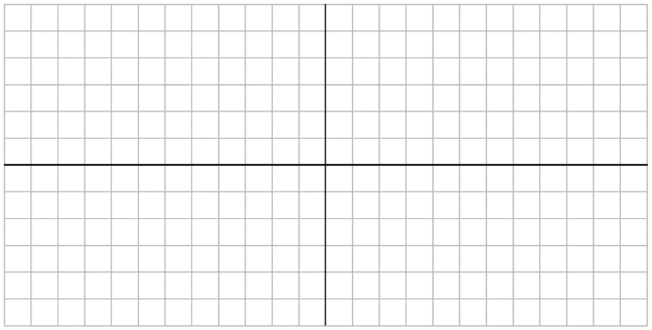
Definition of Amplitude of Sine and Cosine Curves

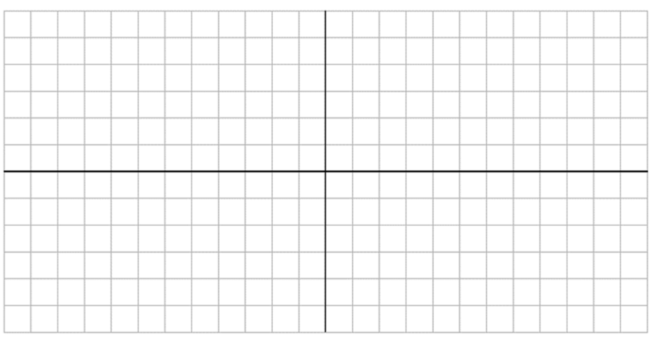
The **amplitude** of $y = a\sin(x)$ and $y = a\cos(x)$ represents half the distance between the maximum and minimum values of the function and is given by

$$\text{amplitude} = |a|.$$

Example 1

Sketch the graph of the functions $y = 3\sin(x)$ and $y = 0.5\cos(x)$ by hand. Identify the amplitude of each graph.





Because $y = a\sin(x)$ completes one cycle from $x = 0$ to $x = 2\pi$, it follows that $y = a\sin(bx)$ completes one cycle from $x = 0$ to $x = 2\pi/b$.

Period of Sine and Cosine Functions

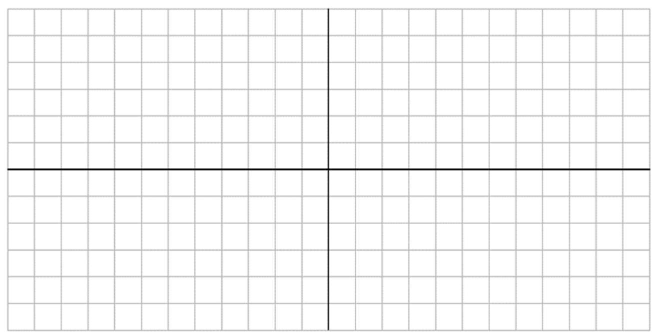
Let b be a positive real number. The **period** of $y = a\sin(bx)$ and $y = a\cos(bx)$ is given by

$$\text{period} = \frac{2\pi}{b}.$$

Note that if $0 < b < 1$, the period of $y = a\sin(bx)$ is greater than 2π and represents a *horizontal stretching* of the graph of $y = a\sin(x)$. Similarly, if $b > 1$, the period of $y = a\sin(bx)$ is less than 2π and represents a *horizontal shrinking* of the graph of $y = a\sin(x)$.

Example 2

Sketch the graph of the function $y = \cos\left(\frac{x}{2}\right)$ by hand. Identify the amplitude and period of the graph.



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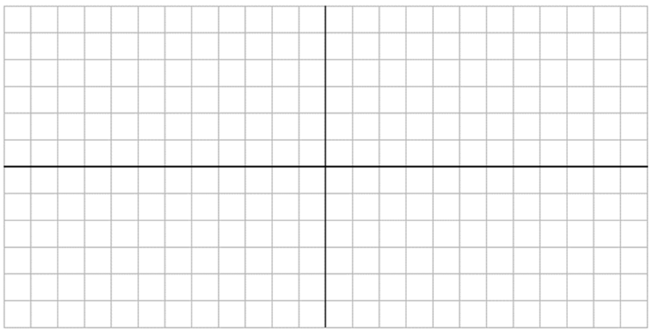
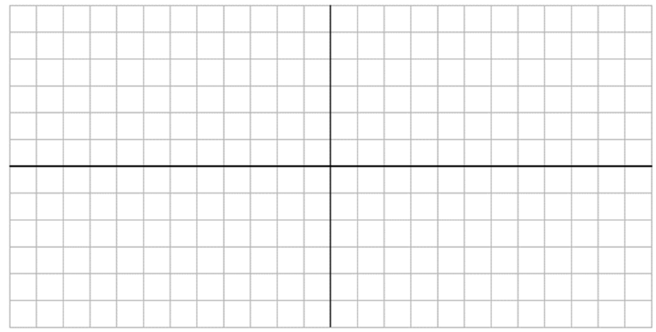
In Exercises 1-4, sketch the graph of the function by hand. (Include a maximum of two full periods.) Identify the amplitude and period of the graph.

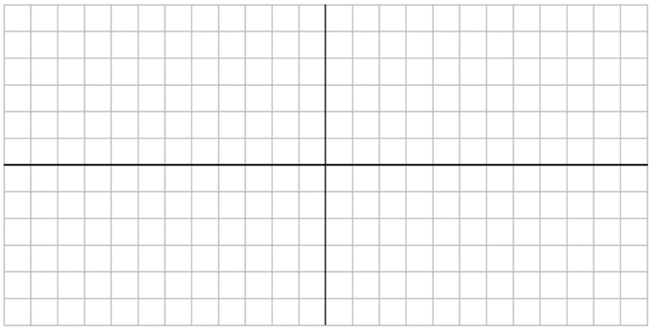
1. $y = 1.5\cos(x)$



2. $y = \sin(2x)$



3. $y = 0.75\sin\left(\frac{4x}{5}\right)$



4. $y = 2.5\cos(3x)$