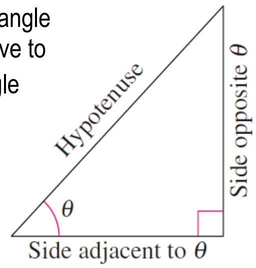
Name ______________________________

Date _________________ Pd ______

The Six Trigonometric Functions

Consider a right triangle, with one acute angle labeled θ, as shown in the figure. Relative to the angle θ, the three sides of the triangle are the **hypotenuse**, the **opposite side** (the side opposite the angle θ), and the **adjacent side** (the side adjacent to the angle θ).



Using the lengths of these three sides, you can form six ratios that defined the six trigonometric functions of the acute angle θ.

Right Triangle Definitions of Trigonometric Functions

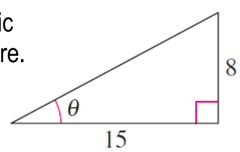
Let θ be an *acute* angle of a right triangle. Then the six trigonometric functions *of the angle* θ are defined as follows. (Note that the functions in the second row are the *reciprocals* of the corresponding functions in the first row.)

$$\sin(\theta)=\frac{\text{opp}}{\text{hyp}} \qquad \cos(\theta)=\frac{\text{adj}}{\text{hyp}} \qquad \tan(\theta)=\frac{\text{opp}}{\text{adj}}$$

$$\csc(\theta)=\frac{\text{hyp}}{\text{opp}} \qquad \sec(\theta)=\frac{\text{hyp}}{\text{adj}} \qquad \cot(\theta)=\frac{\text{adj}}{\text{opp}}$$

Example 1

Find the exact value of the six trigonometric functions of the angle θ shown in the figure. (Use the Pythagorean Theorem to find the third side of the triangle.)



Example 2

Construct an appropriate triangle to evaluate $\sin(30°)$ and $\tan(\pi/3)$.

Sines, Cosines, and Tangents of Special Angles

θ	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$
$30° = \frac{\pi}{6}$ rad	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
$45° = \frac{\pi}{4}$ rad	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$60° = \frac{\pi}{3}$ rad	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

Trigonometric Identities

In trigonometry, a great deal of time is spent studying relationships between trigonometric functions (identities).

Fundamental Trigonometric Identities

Reciprocal Identities

$$\sin(\theta)=\frac{1}{\csc(\theta)} \qquad \cos(\theta)=\frac{1}{\sec(\theta)} \qquad \tan(\theta)=\frac{1}{\cot(\theta)}$$

$$\csc(\theta)=\frac{1}{\sin(\theta)} \qquad \sec(\theta)=\frac{1}{\cos(\theta)} \qquad \cot(\theta)=\frac{1}{\tan(\theta)}$$

Quotient Identities

$$\tan(\theta)=\frac{\sin(\theta)}{\cos(\theta)} \qquad \cot(\theta)=\frac{\cos(\theta)}{\sin(\theta)}$$

Pythagorean Identities

$$\sin^2(\theta)+\cos^2(\theta)=1$$
$$1+\tan^2(\theta)=\sec^2(\theta)$$
$$1+\cot^2(\theta)=\csc^2(\theta)$$

Note: $\sin^2(\theta)$ represents $(\sin(\theta))^2$, $\cos^2(\theta)=(\cos(\theta))^2$, etc.

Example 3

Given $\csc(\theta)=3$ and $\sec(\theta)=\frac{3\sqrt{2}}{4}$, use the trigonometric identities to find the indicated trigonometric functions.

(a) $\sin(\theta)$

(b) $\cos(\theta)$

(c) $\tan(\theta)$

Applications Involving Right Triangles

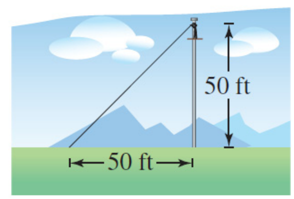
Many applications of trigonometry involve a process called **solving right triangles**. In this type of application, you are usually given:

- one side of a right triangle and one of the acute angles and are asked to find one of the other sides; or
- two sides and are asked to find one of the acute angles.

In the following example, the angle you are given is the **angle of elevation**, which represents the angle from the horizontal upward to the object. In other applications you may be given the **angle of depression**, which represents the angle from the horizontal downward to the object.

Example 4

A zip-line steel cable is being constructed for a reality competition show. The high end of the zip-line is attached to the top of a 50-foot pole while the lower end is anchored at ground level to a stake 50 feet from the base of the pole.

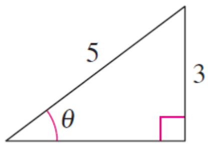


Find the angle of elevation of the zip-line.

Trigonometry/Precalculus — Name ____________________

§4.3 Right Triangle Trigonometry HOMEWORK — Date ____________ Pd ______

1. Find the exact values of the six trigonometric functions of the angle θ shown in the figure. (Use the Pythagorean Theorem to find the third side of the triangle.)



In Exercises 2-3, sketch a right triangle corresponding to the trigonometric function of the acute angle θ. Use the Pythagorean Theorem to determine the third side of the triangle and then find the other five trigonometric functions of θ.

2. $\sin(\theta) = \frac{5}{6}$

3. $\cot(\theta) = \frac{9}{4}$

In Exercises 4-5, construct an appropriate triangle to evaluate the expressions.

4. $\cos(45°)$

5. $\sec(60°)$

In Exercises 6-7, use the given function value(s) and the trigonometric identities to find the indicated trigonometric functions.

6. $\sec(\theta) = 5$, $\tan(\theta) = 2\sqrt{6}$

(a) $\cos(\theta)$

(b) $\cot(\theta)$

(c) $\sin(\theta)$

7. $\cos(\alpha) = 1/4$

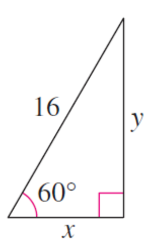
(a) $\sec(\alpha)$

(b) $\sin(\alpha)$

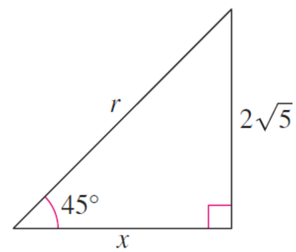
(c) $\cot(\alpha)$

In Exercises 8-9, find the exact values of the indicated variables.

8. Find x and y.



9. Find x and r.



10. A steel plate has the form of one fourth of a circle with a radius of 60 centimeters. Two 2-centimeter holes are to be drilled in the plate, positioned as shown in the figure. Find the coordinates of the center of each hole.