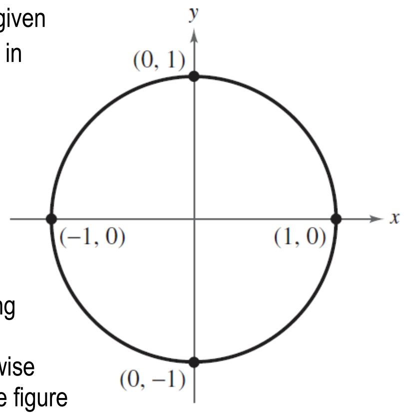
Trigonometry/Precalculus — Name ____________________

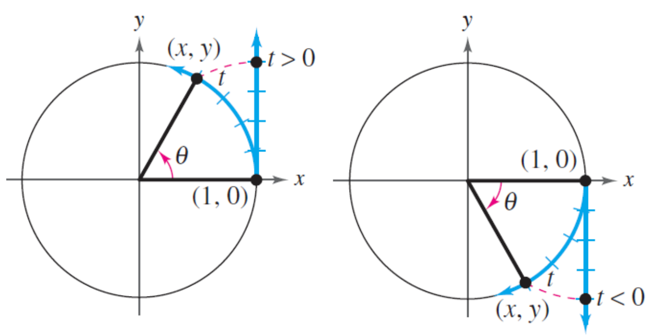
§4.2 Trigonometric Functions: The Unit Circle — Date ______________ Pd ______

The Unit Circle

Consider the **unit circle** given by $x^2 + y^2 = 1$ as shown in the figure.



Imagine that the real number line is wrapped around this circle, with positive numbers corresponding to a counterclockwise wrapping and negative numbers corresponding to a clockwise wrapping, as shown in the figure below.



As the real number line is wrapped around the unit circle, each real number t corresponds to a point (x, y) on the circle. For example, the real number 0 corresponds to the point $(1, 0)$. Moreover, because the unit circle has a circumference of 2π, the real number 2π also corresponds to the point $(1, 0)$.

In general, each real number t also corresponds to a central angle θ (in standard position) whose radian measure is t. With this interpretation of t, the arc length formula $s = r\theta$ (with $r = 1$) indicates that the real number t is the length of the arc intercepted by the angle θ, given in radians.

The Trigonometric Functions

From the preceding discussion, it follows that the coordinates x and y are two functions of the real variable t. You can use these coordinates to define the six trigonometric functions of t.

sine (sin)	**cosecant (csc)**
cosine (cos)	**secant (sec)**
tangent (tan)	**cotangent (cot)**

Definitions of Trigonometric Functions

Let t be a real number and let (x, y) be the point on the unit circle corresponding to t.

$$\sin(t) = y \qquad \csc(t) = \frac{1}{y},\ \ y \neq 0$$

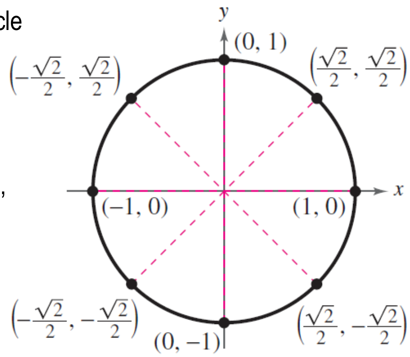
$$\cos(t) = x \qquad \sec(t) = \frac{1}{x},\ \ x \neq 0$$

$$\tan(t) = \frac{y}{x},\ \ x \neq 0 \qquad \cot(x) = \frac{x}{y},\ \ y \neq 0$$

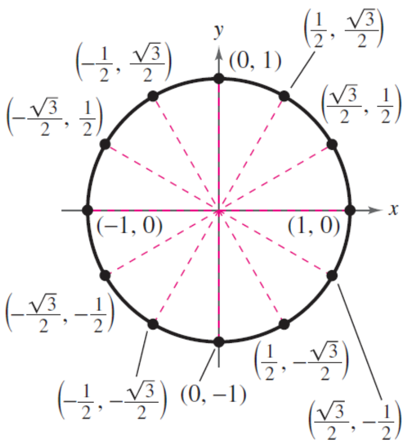
Note that the functions in the second column are the *reciprocals* of the corresponding functions in the first column.

In the definitions of the trigonometric functions, note that the tangent and secant are not defined when $x = 0$. For instance, because $t = \pi/2$ corresponds to $(x, y) = (0, 1)$, it follows that $\tan(\pi/2)$ and $\sec(\pi/2)$ are *undefined*. Similarly, the cotangent and cosecant are not defined when $y = 0$. For instance, because $t = 0$ corresponds to $(x, y) = (1, 0)$, $\cot(0)$ and $\csc(0)$ are *undefined*.

In the figure, the unit circle has been divided into eight equal arcs, corresponding to t-values of $0,\ \pi/4,\ \pi/2,\ 3\pi/4,\ \pi,\ 5\pi/4,\ 3\pi/2,\ 7\pi/4$, and 2π.



Similarly, in this figure, the unit circle has been divided into 12 equal arcs, corresponding to t-values of $0,\ \pi/6,\ \pi/3,\ \pi/2,\ 2\pi/3,\ 5\pi/6,\ \pi,\ 7\pi/6,\ 4\pi/3,\ 3\pi/2,\ 5\pi/3,\ 11\pi/6$, and 2π.



Using the (x, y) coordinates in these two figures, you can easily evaluate the exact values of trigonometric functions for common t-values. This procedure is demonstrated in Examples 1 and 2. You should study and learn these exact values for common t-values because they will help you in later sections to perform calculations quickly and easily.

Example 1

Evaluate (if possible) the six trigonometric functions of the real number $t = 3\pi/4$.

Example 2

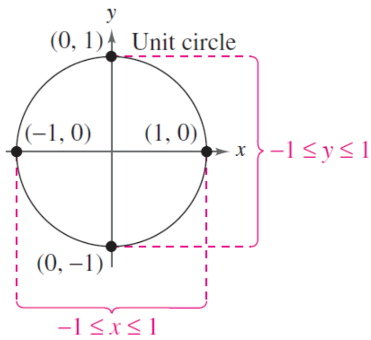
Evaluate (if possible) the six trigonometric functions of the real number $t = -2\pi/3$.

Name ______________________

Date ______________ Pd ____

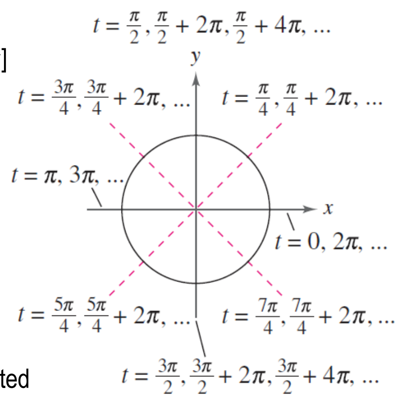
Domain and Period of Sine and Cosine

The *domain* of the sine and cosine functions is the set of all real numbers. To determine the *range* of these two functions, consider the unit circle shown in the figure. Because $r = 1$, it follows that $\sin(t) = y$ and $\cos(t) = x$. Moreover, because (x, y) is on the unit circle, you know that $-1 \le y \le 1$ and $-1 \le x \le 1$. So, the values of sine and cosine also range between -1 and 1.



$$-1 \le y \le 1 \quad \text{and} \quad -1 \le x \le 1$$
$$-1 \le \sin(t) \le 1 \qquad\qquad -1 \le \cos(t) \le 1$$

Adding 2π to each value of t in the interval $[0, 2\pi]$ completes a second revolution around the unit circle, as shown the figure. The values of $\sin(t + 2\pi)$ and $\cos(t + 2\pi)$ correspond to those of $\sin(t)$ and $\cos(t)$. Similar results can be obtained for repeated revolutions (positive or negative) around the unit circle. This leads to the general result



$$\sin(t + 2\pi n) = \sin(t) \quad \text{and} \quad \cos(t + 2\pi n) = \cos(t)$$

for any integer n and real number t. Functions that behave in such a repetitive (or cyclic) manner are called **periodic**.

Definition of a Periodic Function

A function f is **periodic** if there exists a positive real number c such that

$$f(t + c) = f(t)$$

for all t in the domain of f. The least number c for which f is periodic is called the **period** of f.

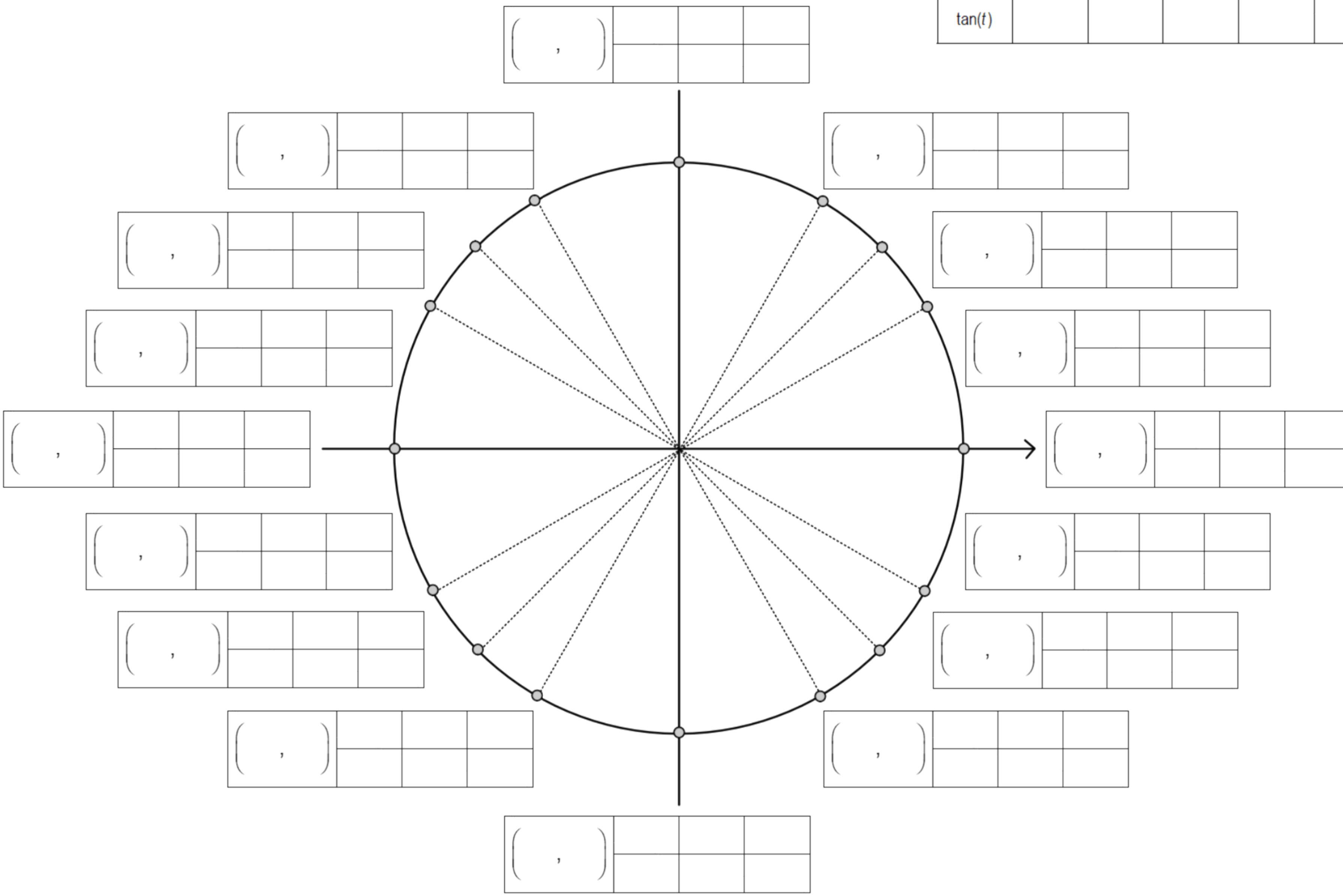
Example 3

Evaluate the trigonometric expression $\cos(8\pi/3)$ using its period as an aid.

Example 4

Use a calculator to evaluate the trigonometric expression $\csc(1.3)$. Leave your answer accurate to four decimal places.

t	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin(t)$					
$\cos(t)$					
$\tan(t)$					



Trigonometry/Precalculus Name ____________________

§4.2 Trigonometric Functions: The Unit Circle HOMEWORK Date ____________ Pd ______

In Exercises 1-6, find the point (x, y) on the unit circle that corresponds to the real number t.

1. $t = \pi/4$

2. $t = 7\pi/6$

3. $t = 2\pi/3$

4. $t = 3\pi/2$

5. $t = -7\pi/4$

6. $t = -3\pi/2$

In Exercises 7-12, evaluate (if possible) the sine, cosine, and tangent of the real number.

7. $t = \pi/4$

8. $t = 7\pi/6$

9. $t = 2\pi/3$

10. $t = -5\pi/3$

11. $t = -\pi/6$

12. $t = -3\pi/2$

In Exercises 13-15, evaluate (if possible) the six trigonometric functions of the real number.

13. $t = 5\pi/6$

14. $t = 3\pi/2$

15. $t = -7\pi/4$

In Exercises 16-18, evaluate the trigonometric function using its period as an aid.

16. $\sin(5\pi)$

17. $\cos(-13\pi/6)$

18. $\sin(-9\pi/4)$

(CA) In Exercises 19-24, use a calculator to evaluate the trigonometric expression. Leave your answer accurate to four decimal places.

19. $\sin(7\pi/9)$

20. $\cos(11\pi/5)$

21. $\csc(0.8)$

22. $\sec(22.8)$

23. $\cot(2.5)$

24. $\tan(1.75)$