Trigonometry/Precalculus

§4.1b Linear and Angular Speed

Name _____

Date _____ Pd _____

Linear and Angular Speed

The *radian measure* formula $\theta = s/r$ can be used to measure arc length along a circle.

Arc Length

For a circle of radius r, a central angle θ intercepts an arc of length s given by

$$s = r\theta$$

where θ is measured in radians. Note that if $r = 1$, then $s = \theta$, and the radian measure of θ equals the arc length.

Example 1

Find the length of the arc on a circle of radius $r = 14$ inches intercepted by a central angle $\theta = 180°$.

The formula for the length of a circular arc can be used to analyze the motion of a particle moving at a *constant speed* along a circular path.

Linear and Angular Speed

Consider a particle moving at a constant speed along a circular arc of radius r. If s is the length of the arc traveled in time t, then the **linear speed** of the particle is

$$\text{linear speed} = \frac{\text{arc length}}{\text{time}} = \frac{s}{t}.$$

Moreover, if θ is the angle (in radian measure) corresponding to the arc length s, then the **angular speed** of the particle is

$$\text{angular speed} = \frac{\text{central angle}}{\text{time}} = \frac{\theta}{t}.$$

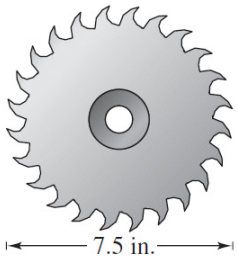
Linear speed measures how fast the particle moves, and angular speed measures how fast the angle changes.

Example 2

A satellite in a circular orbit 1250 kilometers above Earth makes one complete revolution every 110 minutes. What is its linear speed? Assume that Earth is a sphere of radius 6400 kilometers.

Example 3

The circular blade on a saw has a diameter of 7.5 inches (see figure) and rotates at 2400 revolutions per minute.



(a) Find the angular speed in radians per second.

(b) Find the linear speed of the saw teeth (in feet per second) as they contact the wood being cut.

Trigonometry/Precalculus

Name _____

§4.1b Linear and Angular Speed HOMEWORK

Date _____ Pd _____

In Exercises 1-2, find the radian measure of the central angle of a circle of radius r that intercepts an arc of length s.

1. radius $r = 22$ feet, arc length $s = 10$ feet

2. radius $r = 14.5$ centimeters, arc length $s = 35$ centimeters

In Exercises 3-4, find the length of the arc on a circle of radius r intercepted by a central angle θ.

3. radius $r = 9$ feet, central angle $\theta = 60°$

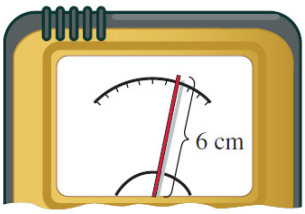
4. radius $r = 27$ meters, central angle $2\pi/3$ radians

In Exercises 5-6, find the radius r of a circle with an arc length s and a central angle θ.

5. arc length $s = 3$ meters, central angle $\theta = 4\pi/3$ radians

6. arc length $s = 82$ miles, central angle $\theta = 135°$

7. A voltmeter's pointer is 6 centimeters in length (see figure). Find the number of degrees through which it rotates when it moves 2.5 centimeters on the scale.



8. The circular blade on a saw has a diameter of 7.25 inches and rotates at 4800 revolutions per minute.

   (a) Find the angular speed of the blade in radians per second.

   (b) Find the linear speed of the saw teeth (in feet per second) as they contact the wood being cut.

9. A computerized spin balance machine rotates a 25-inch diameter tire at 480 revolutions per minute.

   (a) Find the road speed (in miles per hour) at which the tire is being balanced.

   (b) At what rate should the spin balance machine be set so that the tire is being tested for 70 miles per hour?

10. A DVD is approximately 12 centimeters in diameter. The drive motor of the DVD player is controlled to rotate precisely between 200 and 500 revolutions per minute, depending on what track is being read.

    (a) Find an interval for the angular speed of a disc as it rotates.

    (b) Find the linear speed of a point on the outermost track as the disc rotates.