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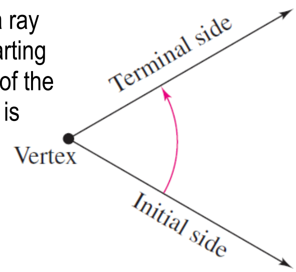
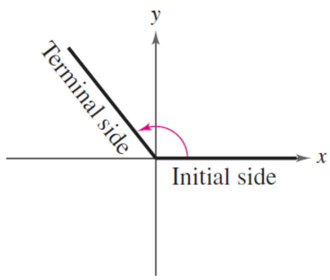
Angles

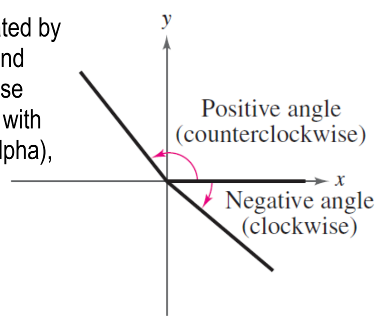
An **angle** is determined by rotating a ray (half-line) about its endpoint. The starting position of the ray is the **initial side** of the angle, and the position after rotation is the **terminal side**. The endpoint of the ray is the **vertex** of the angle.



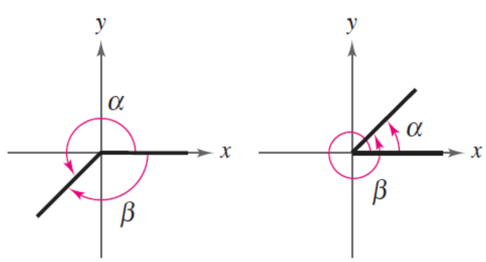
This perception of an angle fits a coordinate system in which the origin is the vertex and the initial side coincides with the positive x-axis. Such an angle is in **standard position**.



Positive angles are generated by counterclockwise rotation, and **negative angles** by clockwise rotation. Angles are labeled with Greek letters such as α (alpha), β (beta), and θ (theta), as well as uppercase letters such as A, B, and C.

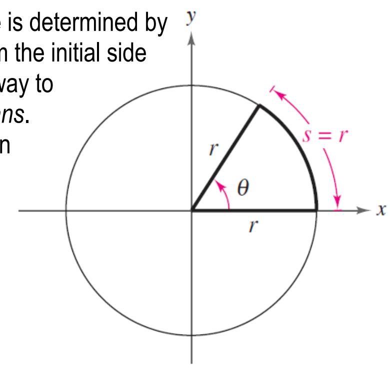


In the figure below, note that angles α and β have the same initial and terminal sides. Such angles are **coterminal**.



Radian Measure

The **measure of an angle** is determined by the amount of rotation from the initial side to the terminal side. One way to measure angles is in *radians*. To define a radian, you can use a **central angle** of a circle, one whose vertex is the center of the circle.

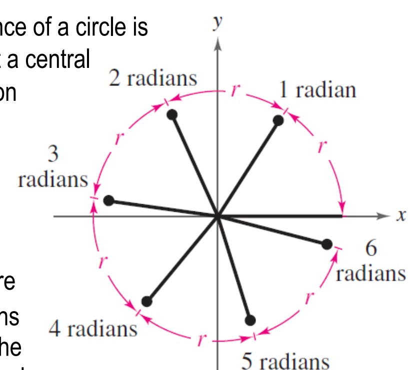


Definition of Radian

One **radian** (rad) is the measure of a central angle θ that intercepts an arc s equal in length to the radius r of the circle.

Algebraically this means that $\theta = \frac{s}{r}$ where θ is measured in radians.

Because the circumference of a circle is $2\pi r$ units, it follows that a central angle of one full revolution (counterclockwise) corresponds to an arc length of $s = 2\pi r$.

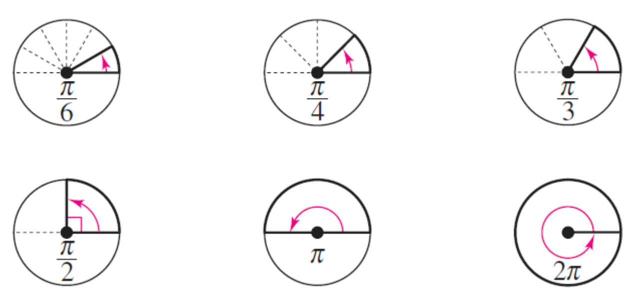


Moreover, because $2\pi \approx 6.283185$, there are just over six radius lengths in a full circle. Because the units of measure for s and r are the same, the ratio s/r has no units—it is simply a real number.

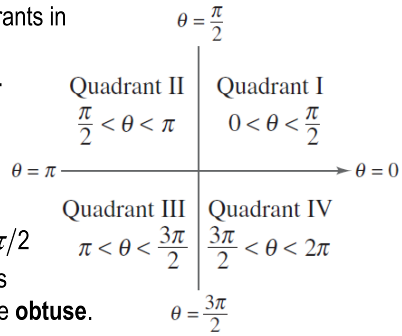
Because the radian measure of an angle of one full revolution is 2π, you can obtain the following.

$$\frac{1}{2}\text{ revolution} = \frac{2\pi}{2} = \pi \text{ rad} \qquad \frac{1}{6}\text{ revolution} = \frac{2\pi}{6} = \frac{\pi}{3}\text{ rad}$$

$$\frac{1}{4}\text{ revolution} = \frac{2\pi}{4} = \frac{\pi}{2}\text{ rad} \qquad \frac{1}{12}\text{ revolution} = \frac{2\pi}{12} = \frac{\pi}{6}\text{ rad}$$



Recall that the four quadrants in a coordinate system are numbered I, II, III, and IV. The figure shows which angles between 0 and 2π lie in each of the four quadrants. Note that angles between 0 and $\pi/2$ are **acute** and that angles between $\pi/2$ and π are **obtuse**.

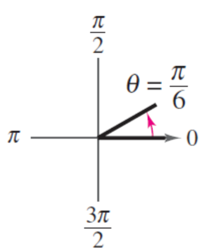


Two angles are coterminal if they have the same initial and terminal sides. For instance, the angles 0 and 2π are coterminal, as are the angles $\pi/6$ and $13\pi/6$. You can find an angle that is coterminal to a given angle θ by adding or subtracting 2π (one revolution). A given angle θ has infinitely many coterminal angles. For instance, $\theta = \pi/6$ is coterminal with $\frac{\pi}{6} + 2n\pi$, where n is an integer.

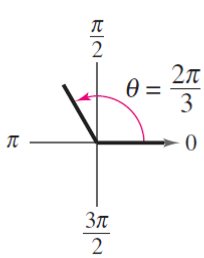
Example 1

Determine two coterminal angles in radian measure (one positive and one negative) for each angle. (There are many correct answers.)

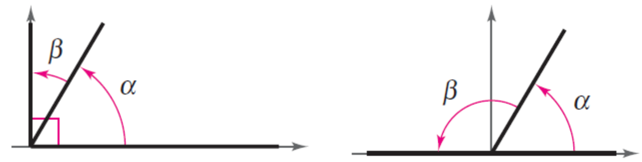
(a)



(b)



Two positive angles α and β are **complementary** (complements of each other) if their sum is $\pi/2$. Two positive angles are **supplementary** (supplements of each other) if their sum is π.



Complementary angles *Supplementary angles*

Example 2

Find (if possible) the complement and supplement of the angle $\pi/3$.

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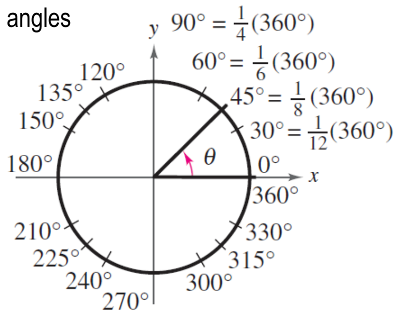
Degree Measure

A second way to measure angles is in terms of **degrees**, denoted by the symbol °. A measure of one degree (1°) is equivalent to a rotation of 1/360 of a complete revolution about the vertex.



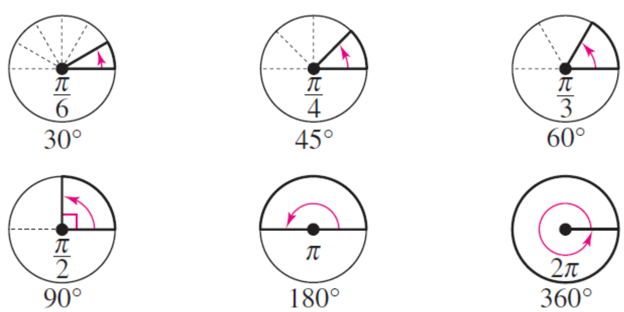
To measure angles, it is convenient to mark degrees on the circumference of a circle, as shown in the figure. A full revolution (counterclockwise) corresponds to 360°, a half revolution to 180°, a quarter revolution to 90°, etc.

Because 2π rad corresponds to one complete revolution, degrees and radians are related by the equations $360° = 2\pi$ rad and $180° = \pi$ rad. From the second equation, you obtain $1° = \frac{\pi}{180}$ rad and $1 \text{ rad} = \frac{180°}{\pi}$ which lead to the following conversion rules.

Conversions Between Degrees and Radians

1. To convert degrees to radians, multiply degrees by $\frac{\pi \text{ rad}}{180°}$.
2. To convert radians to degrees, multiply radians by $\frac{180°}{\pi \text{ rad}}$.

To apply these two conversion rules, use the basic relationship $\pi \text{ rad} = 180°$.



When no units of angle measure are specified, *radian measure is implied*. For instance, if you write $\theta = \pi$ or $\theta = 2$, you imply that $\theta = \pi$ radians or $\theta = 2$ radians.

Example 3

Rewrite each angle in radian measure as a multiple of π. (Do not use a calculator.)

(a) 30°

(b) 150°

Example 4

Rewrite each angle in degree measure. (Do not use a calculator.)

(a) $\frac{3\pi}{2}$

(b) $-\frac{7\pi}{6}$

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In Exercises 1-4, determine the quadrant in which the angle lies. (The angle measure is given in radians.)

1. $\frac{7\pi}{4}$

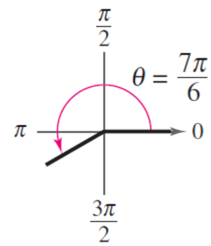
2. $\frac{11\pi}{4}$

3. $-\frac{5\pi}{12}$

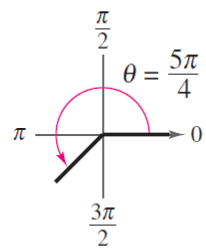
4. $-\frac{13\pi}{9}$

In Exercises 5-8, determine two coterminal angles in radian measure (one positive and one negative) for each angle. (There are many correct answers.)

5.



6.



7. $-\frac{9\pi}{4}$

8. $-\frac{2\pi}{15}$

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In Exercises 9-10, find (if possible) the complement and supplement of the angle.

9. $\frac{\pi}{6}$

10. $\frac{2\pi}{3}$

In Exercises 11-14, rewrite each angle in radian measure as a multiple of π. (Do not use a calculator.)

11. $315°$

12. $120°$

13. $-20°$

14. $-240°$

In Exercises 15-18, rewrite each angle in degree measure. (Do not use a calculator.)

15. $\frac{7\pi}{3}$

16. $-\frac{13\pi}{60}$

17. $-\frac{15\pi}{6}$

18. $\frac{28\pi}{15}$