Trigonometry/Precalculus

§3.4b Solving Logarithmic Equations

Name ______________________

Date ______________ Pd ______

## Introduction

There are two basic strategies for solving exponential or logarithmic equations. The first is based on the One-to-One Properties and the second is based on the Inverse Properties. For $a > 0$ and $a \neq 1,$ the following properties are true for all $x$ and $y$ for which $\log_a(x)$ and $\log_a(y)$ are defined.

| *One-to-One Properties* | *Inverse Properties* |
|---|---|
| $a^x = a^y$ if and only if $x = y$ | $a^{\log_a(x)} = x$ |
| $\log_a(x) = \log_a(y)$ if and only if $x = y$ | $\log_a(a^x) = x$ |

**Strategies for Solving Exponential and Logarithmic Equations**

1. Rewrite the original equation in a form that allows the use of the One-to-One Properties of exponential or logarithmic functions.
2. Rewrite an exponential equation in logarithmic form and apply the Inverse Property of logarithmic functions.
3. Rewrite a logarithmic equation in exponential form and apply the Inverse Property of exponential functions.

## Example 1

Solve each equation.

(a) $\ln(x) - \ln(3) = 0$

(b) $\ln(x) = -3$

(c) $\log_{10}(x) = -1$

## Example 2

Solve each equation. Check your solution(s) algebraically and with a graphing calculator.

(a) $\ln(3x) = 2$

(b) $\log_3(5x - 1) = \log_3(x + 7)$

**Example 3**

Solve each equation. Check your solution(s) algebraically and with a graphing calculator.

(a) $5+2\ln(x)=4$

(b) $\ln(x-2)+\ln(2x-3)=2\ln(x)$

**Example 4**

For selected years from 1985 to 2004, the average salary $y$ (in thousands of dollars) for public school teachers for the year $t$ can be modeled by the equation

$$y=-1.562+14.584\ln(t),\quad 5\leq t\leq 24$$

where $t=5$ represents 1985. During which year did the average salary for public school teachers reach $44,000?

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In Exercises 1-4, solve the logarithmic equation.

1. $\ln(x) - \ln(5) = 0$

2. $\log_{10}(x) = -2$

3. $\log_x(25) = 2$

4. $\ln(3x + 5) = 8$

In Exercises 5-10, solve the logarithmic equation. Check your solution(s) algebraically and with a graphing calculator.

5. $-2 + 2\ln(3x) = 17$

6. $\log_5(3x + 2) = \log_5(6 - x)$

7. $\log_{10}(z - 3) = 2$

8. $\ln(\sqrt{x-8})=5$

9. $\ln(x+5)=\ln(x-1)-\ln(x+1)$

10. $\log_3(x)+\log_3(x-8)=2$

11. Let $N$ represent the number of commercial banks in the United States from 1996 to 2005. $N$ can be modeled by the logarithmic equation

$$N=13{,}387-2190.5\ln(t)$$

where $t$ represents the year, with $t=6$ corresponding to 1996. Use the model to determine during which year the number of banks dropped to 7250.