

Introduction

There are two basic strategies for solving exponential or logarithmic equations. The first is based on the One-to-One Properties and the second is based on the Inverse Properties. For $a > 0$ and $a \neq 1$, the following properties are true for all x and y for which $\log_a(x)$ and $\log_a(y)$ are defined.

One-to-One Properties

$$a^x = a^y \text{ if and only if } x = y$$

$$\log_a(x) = \log_a(y) \text{ if and only if } x = y$$

Inverse Properties

$$a^{\log_a(x)} = x$$

$$\log_a(a^x) = x$$

Strategies for Solving Exponential and Logarithmic Equations

1. Rewrite the original equation in a form that allows the use of the One-to-One Properties of exponential or logarithmic functions.
2. Rewrite an exponential equation in logarithmic form and apply the Inverse Property of logarithmic functions.
3. Rewrite a logarithmic equation in exponential form and apply the Inverse Property of exponential functions.

Example 1

Solve each equation.

(a) $\ln(x) - \ln(3) = 0$

(b) $\ln(x) = -3$

(c) $\log_{10}(x) = -1$

Example 2

Solve each equation. Check your solution(s) algebraically and with a graphing calculator.

(a) $\ln(3x) = 2$

(b) $\log_3(5x - 1) = \log_3(x + 7)$

Example 3

Solve each equation. Check your solution(s) algebraically and with a graphing calculator.

(a) $5 + 2\ln(x) = 4$

(b) $\ln(x - 2) + \ln(2x - 3) = 2\ln(x)$

Example 4

For selected years from 1985 to 2004, the average salary y (in thousands of dollars) for public school teachers for the year t can be modeled by the equation

$$y = -1.562 + 14.584\ln(t), \quad 5 \leq t \leq 24$$

where $t = 5$ represents 1985. During which year did the average salary for public school teachers reach \$44,000?

In Exercises 1-4, solve the logarithmic equation.

1. $\ln(x) - \ln(5) = 0$

2. $\log_{10}(x) = -1$

3. $\log_x(25) = 2$

4. $\ln(3x + 5) = 8$

In Exercises 5-10, solve the logarithmic equation. Check your solution(s) algebraically and with a graphing calculator.

5. $-2 + 2\ln(3x) = 17$

6. $\log_5(3x + 2) = \log_5(6 - x)$

7. $\log_{10}(z - 3) = 2$

8. $\ln(\sqrt{x-8}) = 5$

9. $\ln(x+5) = \ln(x-1) - \ln(x+1)$

10. $\log_3(x) + \log_3(x-8) = 2$

11. Let N represent the number of commercial banks in the United States from 1996 to 2005. N can be modeled by the logarithmic equation

$$N = 13,387 - 2190.5\ln(t)$$

where t represents the year, with $t = 6$ corresponding to 1996. Use the model to determine during which year the number of banks dropped to 7250.