

Change of Base

Most scientific calculators have only two types of log keys, one for common logarithms (base 10) and one for natural logarithms (base e). Although common logarithms and natural logarithms are the most frequently used, you may occasionally need to evaluate logarithms to other bases. To do this, you can use the following **change-of-base formula**.

Change-of-Base Formula

Let a , b , and x be positive real numbers such as $a \neq 1$ and $b \neq 1$. Then $\log_a(x)$ can be converted to a different base using any of the following formulas.

<i>Base b</i>	<i>Base 10</i>	<i>Base e</i>
$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$	$\log_a(x) = \frac{\log_{10}(x)}{\log_{10}(a)}$	$\log_a(x) = \frac{\ln(x)}{\ln(a)}$

Example 1

Rewrite the logarithms $\log_4(25)$ and $\log_2(12)$ as a ratio of (a) common logarithms and (b) natural logarithms.

Properties of Logarithms

Let a be a positive number such that $a \neq 1$, and let n be a real number. If u and v are positive real numbers, the following properties are true.

1. Product Property: $\log_a(uv) = \log_a(u) + \log_a(v)$
2. Quotient Property: $\log_a\left(\frac{u}{v}\right) = \log_a(u) - \log_a(v)$
3. Power Property: $\log_a(u^n) = n\log_a(u)$

Example 2

Write the logarithms $\ln(6)$ and $\ln(2/27)$ in terms of $\ln(2)$ and $\ln(3)$.

Example 3

Use the properties of logarithms to expand the expressions

$$\log_4(5x^3y) \text{ and } \ln\left(\frac{\sqrt{3x-5}}{7}\right).$$

Example 4

Use the properties of logarithms to condense the expressions

$$\frac{1}{2}\log_{10}(x) + 3\log_{10}(x+1) \text{ and } 2\ln(x+2) - \ln(x).$$

In Exercises 1-3, rewrite the logarithm as a ratio of (a) common logarithms and (b) natural logarithms.

1. $\log_3(7)$

2. $\log_9(0.8)$

3. $\log_{20}(135)$

In Exercises 4-6, rewrite the expression in terms of $\ln(4)$ and $\ln(5)$.

4. $\ln(500)$

5. $\ln(5/64)$

6. $\ln(2/5)$

In Exercises 7-9, use the properties of logarithms to rewrite and simplify the logarithmic expression.

7. $\log_4(16)$

8. $\ln(5e^6)$

9. $\ln(6/e^2)$

In Exercises 10-12, expand the expression. Assume all variables are positive.

10. $\log_3(a^2bc^3)$

11. $\ln\left(\frac{xy}{z}\right)$

12. $\ln\left(\sqrt{\frac{x^2}{y^3}}\right)$

In Exercises 13-15, condense the expression to the logarithm of a single quantity.

13. $\ln(x) - 3\ln(x+1)$

14. $2\ln(x) + \ln(x+1)$

15. $\ln(x) - 2[\ln(x+2) - \ln(x-2)]$