Trigonometry/Precalculus

§3.2 Logarithmic Functions and Their Graphs

Name ____________________

Date ____________ Pd ______

Logarithmic Functions

If a function is one-to-one—that is, if the function has the property that no horizontal line intersects its graph more than once—the function must have an inverse function. Every function of the form

$$f(x) = a^x, \quad a > 0, \quad a \neq 1$$

passes the Horizontal Line Test and therefore must have an inverse function. This inverse function is called the **logarithmic function with base *a***.

Definition of Logarithmic Function

For $x > 0$, $a > 0$, and $a \neq 1$,

$$y = \log_a(x) \quad \text{if and only if} \quad x = a^y.$$

The function given by

$$f(x) = \log_a(x) \ \text{(read as "log base } a \text{ of } x\text{")}$$

is called the **logarithmic function with base *a***.

From the definition above, you can see that every logarithmic equation can be written in an equivalent exponential form and every exponential equation can be written in logarithmic form. The equations $y = \log_a(x)$ and $x = a^y$ are equivalent.

Properties of Logarithms

1. $\log_a(1) = 0$ because $a^0 = 1$.
2. $\log_a(a) = 1$ because $a^1 = a$.
3. $\log_a(a^x) = x$ and $a^{\log_a(x)} = x$.
4. If $\log_a(x) = \log_a(y)$, then $x = y$.

Example 1

(a) Solve for x: $\log_2(x) = \log_2(3)$

(b) Solve for x: $\log_4(4) = x$

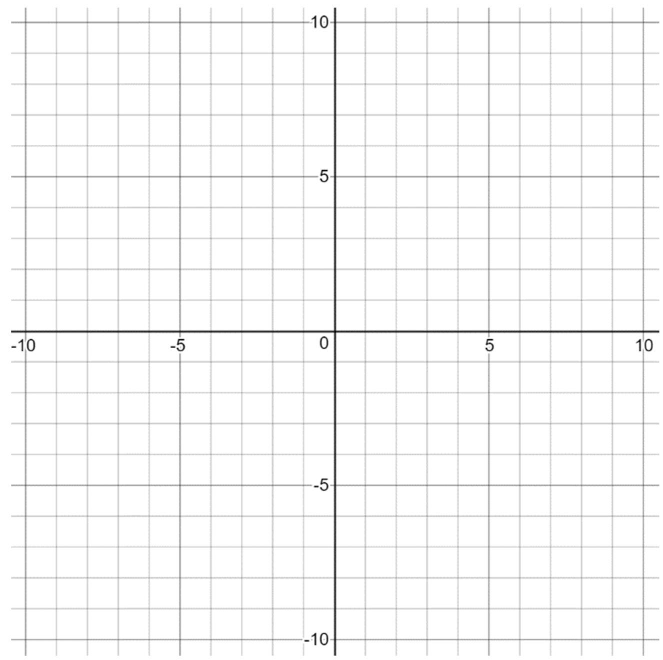
(c) Simplify: $\log_5(5^7)$ and $7^{\log_7(14)}$

Graphs of Logarithmic Functions

To sketch the graph of $y = \log_a(x)$, you can use the fact that the graphs of inverse functions are reflections of each other in the line $y = x$.

Example 2

Sketch the graphs of $f(x) = 2^x$ and $g(x) = \log_2(x)$.



Example 3

Use the graph of $f(x)$ to describe the transformation that yields the graph of $g(x)$

(a) $f(x) = \log_{10}(x)$, $g(x) = \log_{10}(x-1)$

(b) $f(x) = \log_{10}(x)$, $g(x) = \log_{10}(x) + 2$

(c) $f(x) = \log_{10}(x)$, $g(x) = -\log_{10}(x)$

(d) $f(x) = \log_{10}(x)$, $g(x) = \log_{10}(-x)$

(e) $f(x) = \log_{10}(x)$, $g(x) = 4\log_{10}(x)$

(f) $f(x) = \log_{10}(x)$, $g(x) = 0.5\log_{10}(x)$

The Natural Logarithmic Function

For $x > 0$,

$$y = \ln(x) \text{ if and only if } x = e^y.$$

The function given by

$$f(x) = \log_e(x) = \ln(x)$$

is called the **natural logarithmic function**.

Example 4

Use the properties of logarithms to rewrite each expression.

(a) $\ln(1/e)$

(b) $e^{\ln(5)}$

(c) $4\ln(1)$

(d) $2\ln(e)$

Example 5

Find the domain of each function.

(a) $f(x) = \ln(2-x)$

(b) $f(x) = \ln(x^2 - 4)$

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§3.2 Logarithmic Functions and Their Graphs HOMEWORK

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In Exercises 1-4, solve the equation for x.

1. $\log_7(x) = \log_7(9)$

2. $\log_5(5) = x$

3. $\log_6(6^2) = x$

4. $\log_8(x) = \log_8(10^{-1})$

In Exercises 5-8, use the properties of logarithms to rewrite the expression.

5. $6^{\log_6(36)}$

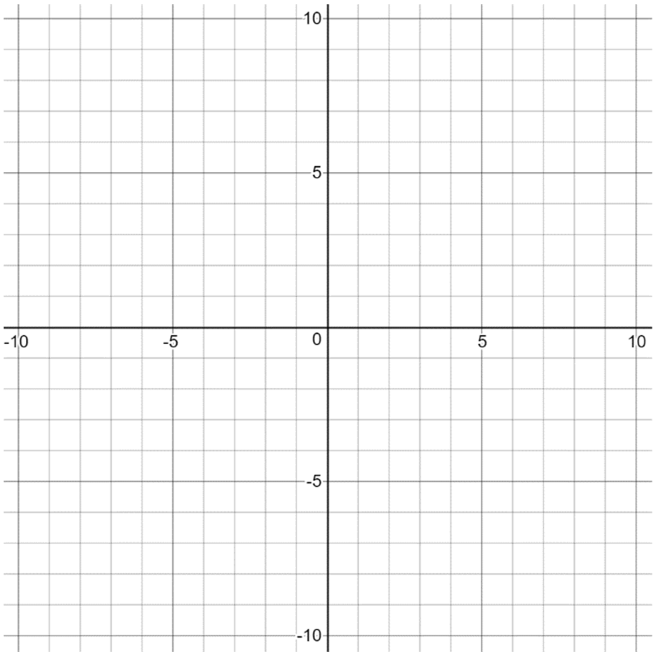
6. $3\log_2(1/2)$

7. $0.25\log_4(16)$

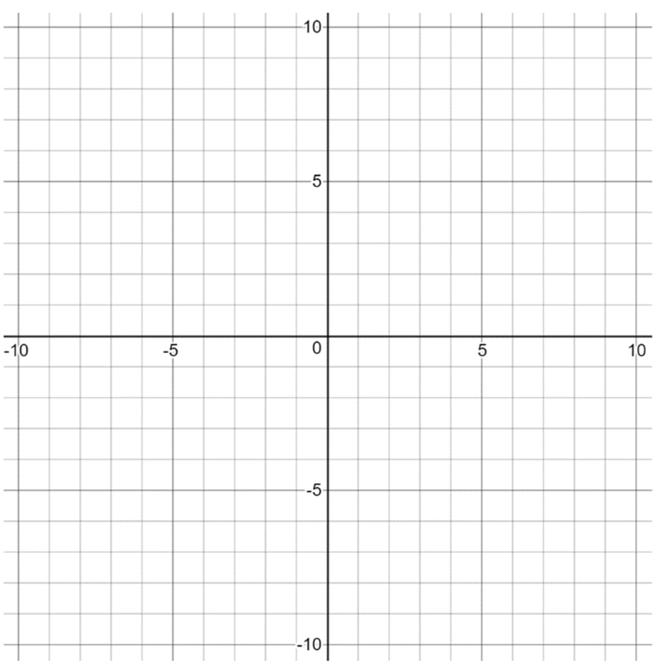
8. $7\ln(e^0)$

In Exercises 9-10, sketch the graph of $f(x)$. Then use the graph of $f(x)$ to sketch the graph of $g(x)$.

9. $f(x) = 3^x$, $g(x) = \log_3(x)$



10. $f(x) = (1/2)^x$, $g(x) = \log_{1/2}(x)$



In Exercises 11-16, use the graph of $f(x)$ to describe the transformation that yields the graph of $g(x)$

11. $f(x) = \log_{10}(x)$, $g(x) = -\log_{10}(x)$

12. $f(x) = \log_{10}(x)$, $g(x) = \log_{10}(x+7)$

13. $f(x) = \log_2(x)$, $g(x) = 4 - \log_2(x)$

14. $f(x) = \log_2(x)$, $g(x) = 3 + 4\log_2(x)$

15. $f(x) = \ln(x)$, $g(x) = -\ln(x+3)$

16. $f(x) = \ln(x)$, $g(x) = \ln(x+2) - 5$