

Definition of Exponential Function

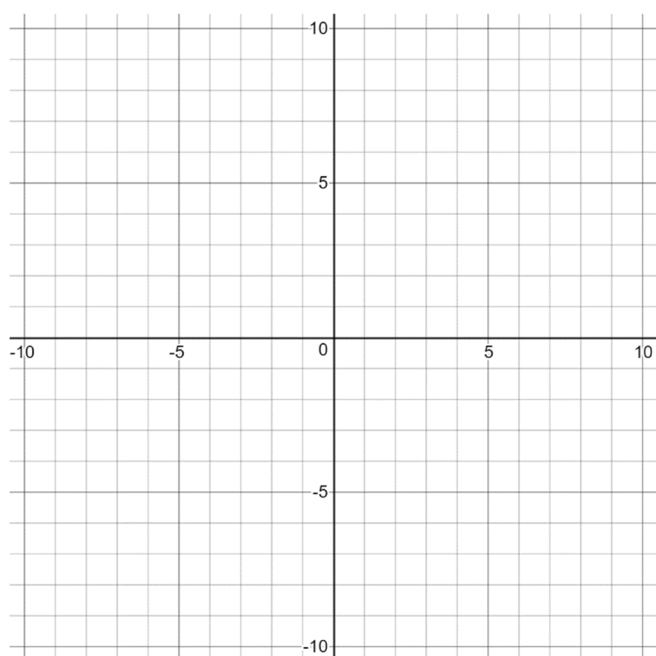
The **exponential function f with base a** is denoted by

$$f(x) = a^x$$

where $a > 0$, $a \neq 1$, and x is any real number.

Example 1

Sketch the graphs of $y = 2^x$, $y = 3^x$, $y = 2^{-x}$, and $y = 3^{-x}$ by hand. Verify your graphs with a graphing calculator.

**Example 2**

Use the graph of $f(x)$ to describe the transformation that yields the graph of $g(x)$.

(a) $f(x) = 3^x$, $g(x) = 3^{x+1}$

(b) $f(x) = 3^x$, $g(x) = 3^x - 2$

(c) $f(x) = 3^x$, $g(x) = -3^x$

(d) $f(x) = 3^x$, $g(x) = 3^{-x}$

(e) $f(x) = 3^x$, $g(x) = 2 \cdot 3^x$

(f) $f(x) = 3^x$, $g(x) = 0.25 \cdot 3^x$

The Natural Base e

For many applications, the convenient choice for a base is the irrational number $e \approx 2.718281828$. This number is called the **natural base**. The function $f(x) = e^x$ is called the **natural exponential function**.

Formulas for Compound Interest

After t years, the balance A in an account with principal P and annual interest rate r (in decimal form) is given by the following formulas.

1. For n compoundings per year: $A = P \left(1 + \frac{r}{n} \right)^{nt}$
2. For continuous compounding: $A = Pe^{rt}$

Example 3

A total of \$12,000 is invested at an annual interest rate of 3%. Find the balance after 4 years if the interest is compounded (a) quarterly and (b) continuously.

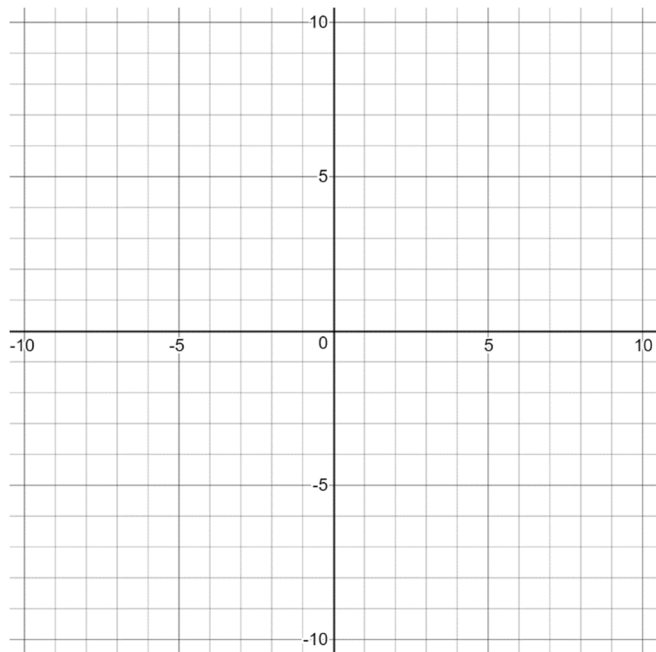
Example 4

Let y represent a mass, in grams, of radioactive strontium (^{90}Sr), whose half-life is 29 years. The initial mass of strontium present is 10 grams. How much strontium is present after 80 years?

Example 5

The approximate number of fruit flies in an experimental population after t hours is given by $Q(t) = 20e^{0.03t}$, where $t \geq 0$. How large is the population of fruit flies after 72 hours?

In Exercises 1-4, sketch the graph of the exponential function. Verify your graphs with a graphing calculator.



1. $y = e^x$

2. $y = 0.5^x$

3. $y = 2^x - 7$

4. $y = -3^x + 8$

In Exercises 5-10, use the graph of f to describe the transformation that yields the graph of g .

5. $f(x) = 3^x$, $g(x) = 3^{x-5}$

6. $f(x) = -2^x$, $g(x) = 5 - 2^x$

7. $f(x) = 0.6^x$, $g(x) = 3 \cdot 0.6^x$

8. $f(x) = 0.3^x$, $g(x) = -0.3^x + 5$

9. $f(x) = 4^x$, $g(x) = 4^{x-2} - 3$

10. $f(x) = 0.5^x$, $g(x) = 0.5^{-x-4}$

In Exercises 11-12, determine the balance A for P dollars invested at rate r for t years and compounded n times per year.

11. $P = \$2500$, $r = 2.5\%$, $t = 10$ years, $n = 4$

12. $P = \$1000$, $r = 6\%$, $t = 20$ years, $n = 12$

In Exercises 13-14, determine the balance A for P dollars invested at rate r for t years, compounded continuously.

13. $P = \$4000$, $r = 4\%$, $t = 5$ years

14. $P = \$12,000$, $r = 3.5\%$, $t = 30$ years

15. Let Q represent a mass, in grams, of carbon 14 (^{14}C), whose half-life is 5715 years. The initial mass of carbon 14 present is 20 grams. How much carbon 14 is present after 2000 years?

16. In early 2006, a new Jeep Wrangler Sport Edition had a manufacturer's suggested retail price of \$23,790. After t years, the Jeep's value is given by $V(t) = 23,970(0.75)^t$. What is the Jeep's value in 2026?