

Slant Asymptotes

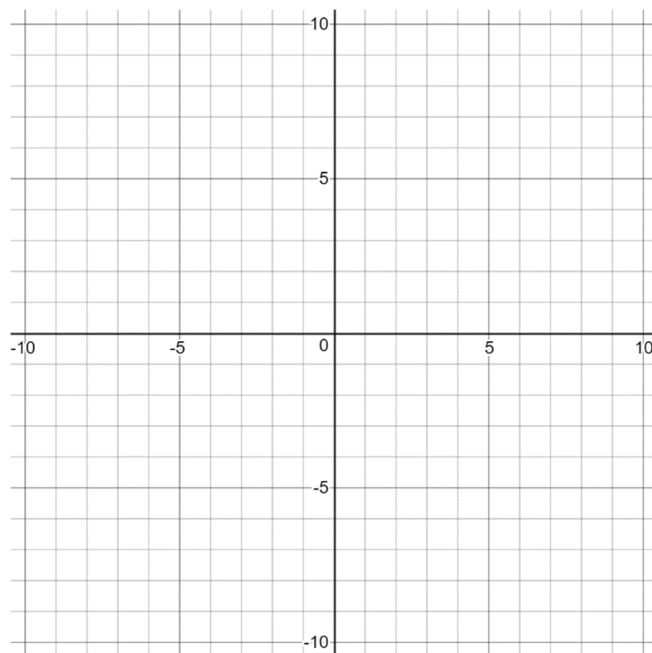
Consider a rational function whose denominator is of degree 1 or greater. If the degree of the numerator is exactly *one more* than the degree of the denominator, the graph of the function has a **slant (or oblique) asymptote**. To find the equation of a slant asymptote, use long division.

Example 1

Find the slant asymptote of the graph of $f(x) = \frac{x^2 - x - 2}{x - 1}$.

Example 2

Sketch the graph of $f(x) = \frac{x}{x^2 - x - 2}$.

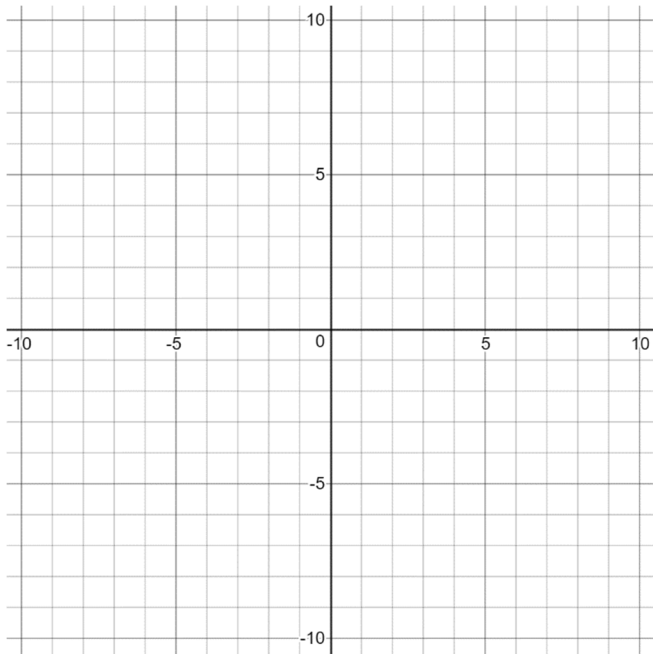
**Guidelines for Graphing Rational Functions**

Let $f(x) = N(x)/D(x)$, where $N(x)$ and $D(x)$ are polynomials.

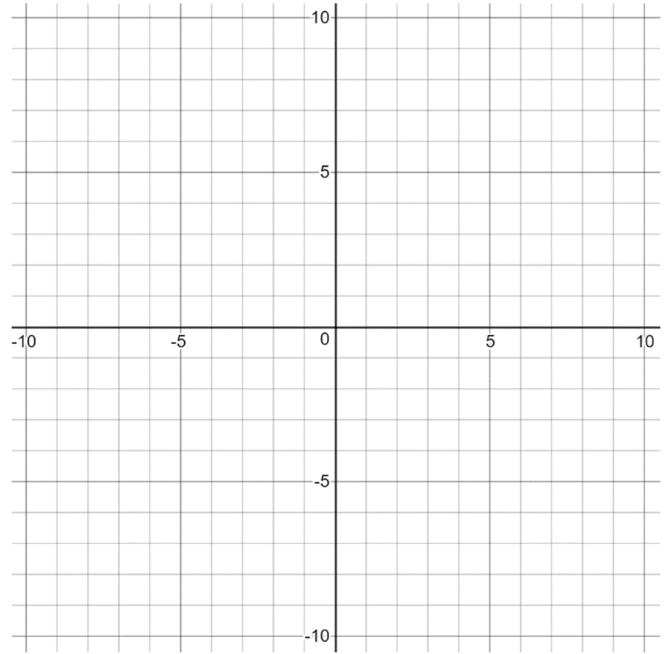
1. Simplify f , if possible.
2. Find and plot the y -intercept (if any) by evaluating $f(0)$.
3. Find the zeros of the numerator (if any) by setting the numerator equal to zero. Then plot the corresponding x -intercepts.
4. Find the zeros of the denominator (if any) by setting the denominator equal to zero. Then sketch the corresponding vertical asymptotes using dashed vertical lines and plot the corresponding holes using open circles.
5. Find and sketch any other asymptotes (horizontal or slant) of the graph using dashed lines.
6. Plot a point to the left and right of each vertical asymptote, and at the smallest and largest x -values on the grid.
7. Use smooth curves to complete the graph between and beyond the vertical asymptotes, excluding any points where f is not defined.

Example 3

Sketch the graph of $f(x) = \frac{x^2 - 9}{x^2 - 2x - 3}$.

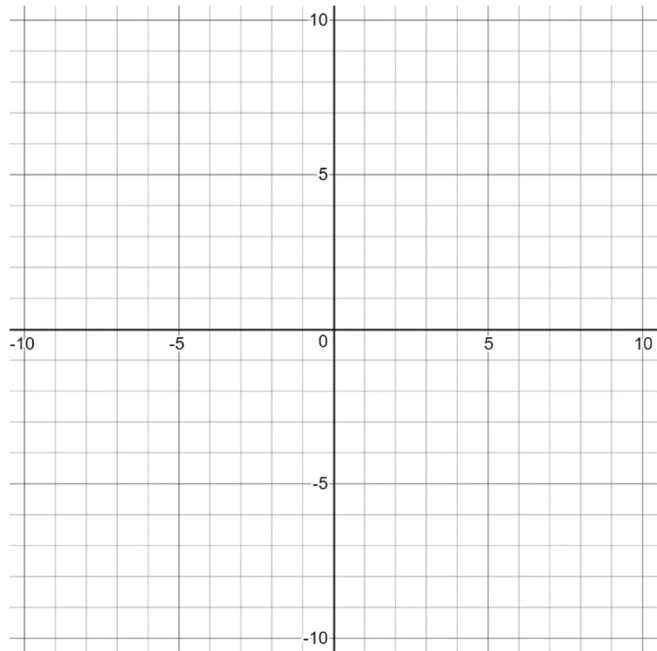
**Example 4**

Sketch the graph of $f(x) = \frac{x^2 - x - 2}{x - 1}$.

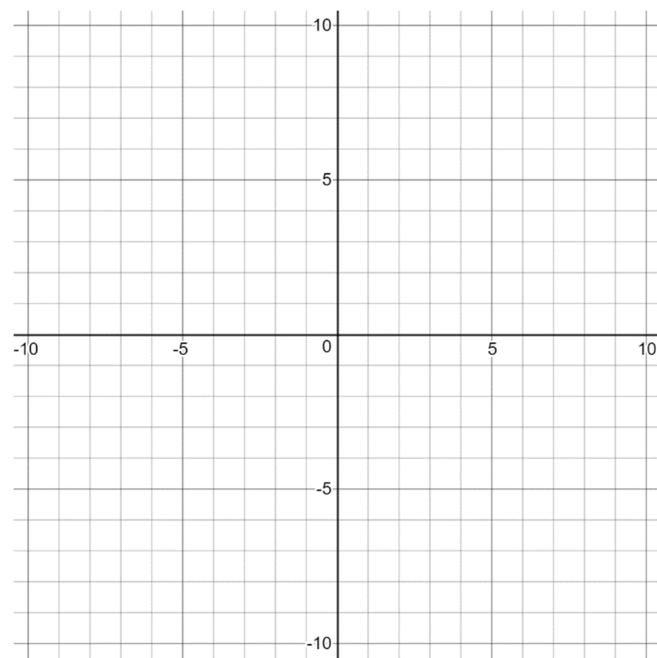


In Exercises 1-4, sketch the graph of the function.

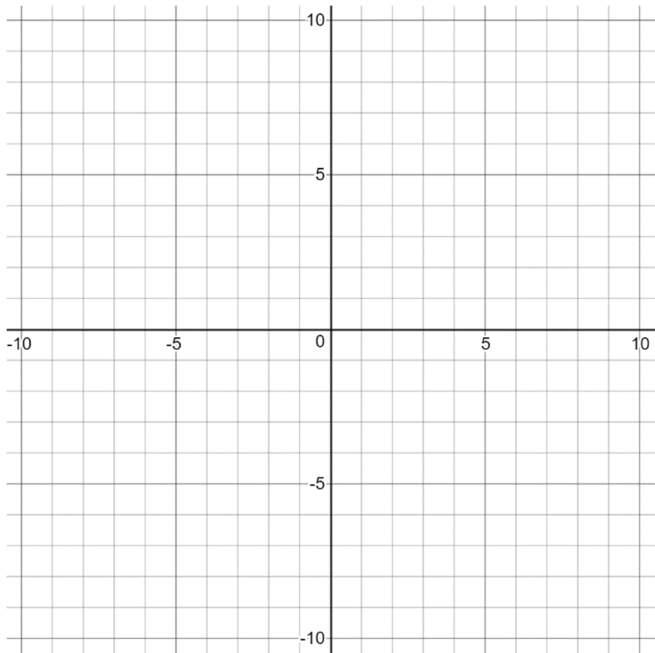
1. $f(x) = \frac{1}{x-6}$



2. $f(x) = \frac{x^2}{x^2 - 4}$



3. $f(x) = \frac{x^2 + 3x}{x^2 + x - 6}$



4. $f(x) = \frac{x^3}{2x^2 - 8}$

