Directed Study in Mathematics | Name ______________________

3.7 Arc Length of a Polar Curve | Date ____________ Pd ____

Required reading:

- Larson 9e: page 745
- Dawkins: Calculus II, section 3-9: Arc Length with Polar Coordinates
  http://tutorial.math.lamar.edu/Classes/CalcII/PolarArcLength.aspx
  - Notes: Read all. (Last modified: 05/31/2018)
  - Practice Problems: Review all. (Last modified: 06/04/2018)

Required homework:

- Larson 9e: page 748, problems 55, 56, 57, 58, 59, 60
- Dawkins: Assignment Problems 4, 6 (Last modified: 03/19/2018)

Additional comments regarding the Larson reading:

Example 4 shows the use of a trigonometric identity to rewrite line 3 as line 4, but an alternate strategy is to use an algebraic technique related to conjugates:

$$
\begin{aligned}
2\sqrt{2}\int_0^{2\pi}\sqrt{1-\cos(\theta)}\,d\theta &= 2\sqrt{2}\int_0^{2\pi}\frac{\sqrt{1-\cos(\theta)}}{1}\cdot\frac{\sqrt{1+\cos(\theta)}}{\sqrt{1+\cos(\theta)}}\,d\theta \\
&= 2\sqrt{2}\int_0^{2\pi}\frac{\sqrt{1-\cos^2(\theta)}}{\sqrt{1+\cos(\theta)}}\,d\theta \\
&= 2\sqrt{2}\int_0^{2\pi}\frac{\left|\sin(\theta)\right|}{\sqrt{1+\cos(\theta)}}\,d\theta \\
&= 2\sqrt{2}\cdot 2\int_0^{\pi}\frac{\sin(\theta)}{\sqrt{1+\cos(\theta)}}\,d\theta
\end{aligned}
$$

Here, we use the fact that $\sin^2(\theta)+\cos^2(\theta)=1$ to rewrite $\sqrt{1-\cos^2(\theta)}$ as $\sqrt{\sin^2(\theta)}=\left|\sin(\theta)\right|$. Note that, for $0\le\theta\le 2\pi$, $\left|\sin(\theta)\right|=\sin(\theta)$, so we can use that fact and symmetry to rewrite the integral in the last line. From there, we can substitute with $u=1+\cos(\theta)$:

$$
\begin{aligned}
4\sqrt{2}\int_0^{\pi}\frac{\sin(\theta)}{\sqrt{1+\cos(\theta)}}\,d\theta &= -4\sqrt{2}\int_2^{0}u^{-1/2}\,du \\
&= \left[-8\sqrt{2u}\,\right]_{u=2}^{u=0} \\
&= -8\sqrt{2\cdot 0}+8\sqrt{2\cdot 2} \\
&= 16
\end{aligned}
$$

Additional comments regarding the Dawkins reading:

Example 1 requires the use of a trigonometric substitution, which is a separate topic in quarter 3.