Directed Study in Mathematics | Name ______________________

1.8 Numerical Integration | Date ______________ Pd ______

Required reading:

- Larson 9e: pages 311-314
- Dawkins: Calculus II, section 1-10: Approximating Definite Integrals
  http://tutorial.math.lamar.edu/Classes/CalcII/ApproximatingDefIntegrals.aspx
  - Notes: Read from the beginning through the table following Example 1. (Last modified: 05/30/2018)
  - Practice Problems: Review parts (b) and (c) of Problems 1-3. (Last modified: 06/04/2018)

Required homework:

- Larson 9e: page 316, problems 4, 5, 7, 8

Additional comments regarding the Larson reading:

In AP Calculus BC, we studied different ways of approximating the value of definite integrals—we used Riemann sums and trapezoidal approximations. This lesson is an extension of that topic.

The Trapezoidal Rule is a method for using trapezoidal approximations specifically where the subintervals are of an equal width; we did not cover this rule in AP Calculus BC because we often had subintervals that were NOT of equal width. On page 311, Larson explains how the rule is developed; here is a more explicit explanation:

$$\begin{aligned}\text{Area} &= \frac{b-a}{n}\cdot\frac{f(x_0)+f(x_1)}{2}+\frac{b-a}{n}\cdot\frac{f(x_1)+f(x_2)}{2}+\frac{b-a}{n}\cdot\frac{f(x_2)+f(x_3)}{2}+\cdots+\frac{b-a}{n}\cdot\frac{f(x_{n-2})+f(x_{n-1})}{2}+\frac{b-a}{n}\cdot\frac{f(x_{n-1})+f(x_n)}{2}\\ &= \frac{b-a}{n}\left(\frac{f(x_0)+f(x_1)}{2}+\frac{f(x_1)+f(x_2)}{2}+\frac{f(x_2)+f(x_3)}{2}+\cdots+\frac{f(x_{n-2})+f(x_{n-1})}{2}+\frac{f(x_{n-1})+f(x_n)}{2}\right)\\ &= \frac{b-a}{n}\cdot\frac{1}{2}\left(f(x_0)+f(x_1)+f(x_1)+f(x_2)+f(x_2)+f(x_3)+\cdots+f(x_{n-2})+f(x_{n-1})+f(x_{n-1})+f(x_n)\right)\\ &= \frac{b-a}{2n}\left(f(x_0+2f(x_1)+2f(x_2)+2f(x_3)+\cdots+2f(x_{n-2})+2f(x_{n-1})+f(x_n)\right)\end{aligned}$$

Note how, with each trapezoid, every base is used twice (once as a second base, again as a first base) except the very first base and the very last base. That is why all of the interior bases are doubled, while the initial and final base are not.

Simpson's Rule is another method that requires subintervals of equal width, with the added requirement that an even number of subintervals be used. Rather than use rectangles or trapezoids to approximate the region under a curve, we use quadratic functions to approximate the curve, with a different quadratic approximation for each pair of adjacent subintervals.

For any given pair of subintervals $[x_0, x_1]$ and $[x_1, x_2]$, for a function $f(x)$, we have three points: $(x_0, f(x_0))$, $(x_1, f(x_1))$, and $(x_2, f(x_2))$. We can then find a quadratic function of the form $p(x) = Ax^2 + Bx + C$ that goes through all three of these points:

$$\begin{aligned}f(x_0) &= Ax_0^2 + Bx_0 + C\\ f(x_1) &= Ax_1^2 + Bx_1 + C\\ f(x_2) &= Ax_2^2 + Bx_2 + C\end{aligned}$$

This is a system of equations, with three equations for three unknowns A, B, and C. This system can be solved for those unknown, determining a unique quadratic function that goes through those three points. We then use the formula below (proven on page 313) to determine the area of the region between the graph of that quadratic function and the x-axis on a closed interval $[a, b]$.

$$\int_a^b p(x)\,dx = \frac{b-a}{6}\left[p(a)+4p\left(\frac{a+b}{2}\right)+p(b)\right]$$