Directed Study in Mathematics | Name ______________________

1.7 Summation Formulas and Definite Integrals | Date ____________ Pd ______

<u>Required reading:</u>

- Larson 9e: pages 259-267 (read all), 271-274 (read from the beginning through Example 2)
- Dawkins: Calculus I, section 7-8: Summation Notation
  http://tutorial.math.lamar.edu/Classes/CalcI/SummationNotation.aspx
  - Notes: Read all. (Last modified: 05/30/2018)
- Dawkins: Calculus I, section 5-5: Area Problem
  http://tutorial.math.lamar.edu/Classes/CalcI/AreaProblem.aspx
  - Notes: Skim all. Read from "Now, let's move on" through "In other words." (Last modified: 05/30/2018)
- Dawkins: Calculus I, section 5-6: Definition of the Definite Integral
  http://tutorial.math.lamar.edu/Classes/CalcI/DefnofDefiniteIntegral.aspx
  - Notes: Read from the beginning through Example 1. (Last modified: 05/30/2018)
  - Practice Problems: Review Problems 1-2. (Last modified: 02/26/2018)

<u>Required homework:</u>

- Larson 9e: page 269, problems 57, 64, 71 (it is not necessary to sketch the region)
- Larson 9e: page 278, problems 4, 8
- Dawkins: Section 5-6, Assignment problems 1, 2, 4 (Last modified: 02/25/2018)

<u>Additional comments regarding this topic:</u>

One of the main purposes of this topic is to see how we can use summations to evaluate Riemann sums and definite integrals. Pay close attention to any time when we do this.

<u>Additional comments regarding the Larson reading:</u>

When evaluating summations like that in page 260, Example 2, be careful with the algebra. In this problem, i is the index of summation, and n represents an arbitrary constant as the upper bound of summation. As such, we can factor out n, but we cannot factor out i.

The discussion on page 261 about inscribed and circumscribed polygons is meant to lead into Example 3 on page 262 and the discussion of upper and lower sums on page 263. The actual area of a region has to be between the smaller (lower sum) and larger (upper sum) areas. Then, in Example 4 and the discussion that follows, we invoke the Squeeze Theorem to show that it does not really matter where we make our choice for x in each subinterval. That realization will come into play later, when Larson defines the Riemann sum on page 272.

In Example 1 on page 271, Larson does not use subintervals of equal width, like he did in previous examples. Instead, we must determine the width of the ith subinterval by subtracting the x-value of the left endpoint from the x-value of the right endpoint. Also note that, usually, $\sqrt{i^2} = |i|$ and $\sqrt{n^2} = |n|$. But since i and n both represent positive quantities, we can omit the absolute values in our algebra.

On page 272, Larson brings up a good point that $n \to \infty$ does not imply that $\|\Delta\| \to 0$. That is why, in his definition of definite integral on page 273, he uses $\lim\limits_{\|\Delta\| \to 0}$ and not $\lim\limits_{n \to \infty}$. Once we establish that we will use a regular partition, and not something unique and troublesome like that mentioned on page 272, then we can replace $\|\Delta\| \to 0$ with $n \to \infty$ and proceed with our computations (see Example 2 on page 274).

The start of the section "Definite Integrals" on page 273 makes a passing reference to the epsilon-delta definition of limit. We will not concern ourselves with this in the current lesson.

We saw the statement, "continuity implies integrability" in AP Calculus BC. We also saw that continuity is not a requirement for integrability—a piecewise-defined function with at least one jump discontinuity but no infinite discontinuities on a closed interval is also integrable.

Additional comments regarding the Dawkins reading:

Section 7-8 is assigned so we can see an additional example utilizing summation formulas. Dawkins assumes we saw this content in Precalculus (hence it is in his "Extras" chapter), though we do not cover this content in our Precalculus course as in-depth as he does.

We can simply skim through most of section 5-5 because it should be familiar to us already—in the beginning, Dawkins is dealing with right, left, and midpoint Riemann sums (though he does not define the term "Riemann sum" until *after* Example 1). At the end, Dawkins discusses the concept of signed area. These portions of the reading are assigned just so we do not jump straight into the middle of his "conversation."

The middle part of the reading for section 5-5 is important, for it leads directly into the start of section 5-6.

In section 5-6, Dawkins defines the definite integral with two differences, compared to Larson. Dawkins uses subintervals of equal width and $\lim_{n\to\infty}$, while Larson does not specify how to partition and uses $\lim_{\|\Delta\|\to 0}$. These definitions are not contradictory—since Dawkins specifies that the subintervals are of equal width, the partition must therefore be regular, and so we can safely replace $\|\Delta\|\to 0$ with $n\to\infty$. In fact, Dawkins' definition of definite integral is almost identical to Larson's definition of the area of a region in the plane (page 265), except Larson includes the requirement that f be nonnegative in the interval in question.

Dawkins is a bit sloppy with his notation of definite integrals. When the integrand involves a sum or difference, he does not consistently put parentheses around it (see section 5-6, Example 1).

In section 5-6, Example 1, Dawkins opts to use the right endpoint for setting up his Riemann sum. Recall that this is not a defined rule—he could easily use a left endpoint. In that case, the Riemann sum would be $\sum_{i=1}^{n} f\left(\frac{2(i-1)}{n}\right)\left(\frac{2}{n}\right)$. This does make the algebra a little more complicated, which is probably why a right endpoint is a better idea here.