

Required reading:

- Larson 9e: pages 229-232
- Dawkins: Calculus I, section 4-13: Newton's Method
<http://tutorial.math.lamar.edu/Classes/CalcI/NewtonsMethod.aspx>
 - Notes: Read all. (Last modified: 05/30/2018)
 - Practice Problems: Review all. (Last modified: 02/21/2018)

Required homework:

- Larson 9e: page 233, problems 7, 8, 14
- Dawkins: Assignment problems 2, 8, 12 (Last modified: 02/21/2018)

Additional comments regarding this topic:

As we proceed with learning about this process, we will see that use of technology will likely be important. There are numerous calculations to perform, and performing them with technology will make it much easier.

Some of these examples, such as those by Larson, utilize tables to organize information. This can be very useful, but also very time consuming to write out step-by-step. Ultimately, when checking student work for accuracy, we simply look at the values of x_n to make sure we obtained the correct values. If there are ways to find those values using technology in an easier way, we should take advantage of them.

Two different ways are with a graphing calculator and with a spreadsheet program (such as Microsoft Excel or Google Sheets). At the end of these notes, we provide some screenshots for the following example: Use Newton's Method to approximate a positive solution to $x^2 - 2 = 0$, accurate to four decimal places. (We know, algebraically, the solution is $x = \sqrt{2} \approx 1.4142$.)

Additional comments regarding the Larson reading:

The algebra used to obtain the equation $x = x_1 - \frac{f(x_1)}{f'(x_1)}$ is fairly simple, but we show it here to be explicit:

$y = f'(x_1)(x - x_1) + f(x_1)$	Original equation: tangent line at $x = x_1$
$0 = f'(x_1)(x - x_1) + f(x_1)$	Let $y = 0$.
$-f(x_1) = f'(x_1)(x - x_1)$	Subtract $f(x_1)$ from both sides.
$-\frac{f(x_1)}{f'(x_1)} = x - x_1$	Divide both sides by $f'(x_1)$.
$x_1 - \frac{f(x_1)}{f'(x_1)} = x$	Add x_1 to both sides.

In Step 3 of the Newton's Method process, $|x_n - x_{n+1}|$ is the difference in magnitude between the old estimate and the new estimate. This difference in magnitude might be relatively large for early iterations of Newton's Method, but as we continue the process, the estimates will improve in quality, and subsequent differences in magnitude will get quite small.

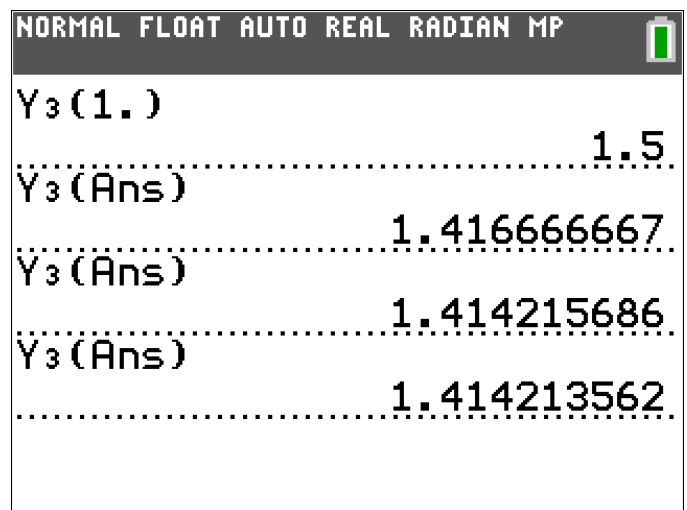
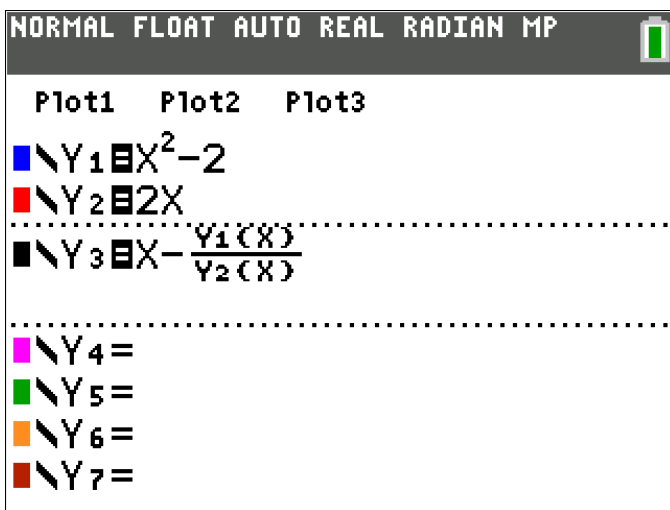
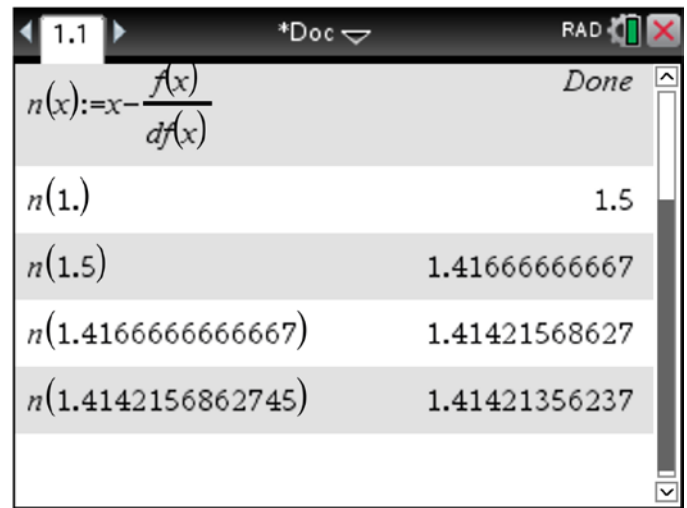
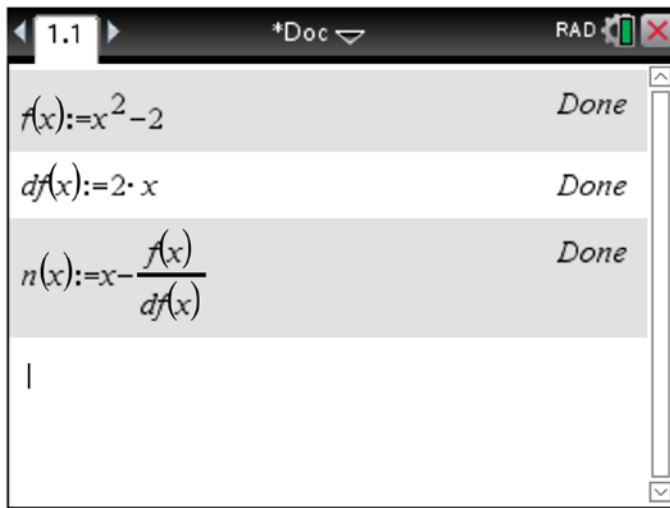
Sometimes we only repeat the process for a fixed number of iterations—we see this in Example 1. Otherwise, we may continue until this difference in magnitude is so small that we decide to stop, because subsequent iterations will not give us improved estimates. This is what happens in Example 2.

At the end of the reading, Larson mentions that there are algebraic ways to find roots of polynomial equations. We know the Quadratic Formula (for finding roots of quadratic equations), but there is also a formula to find roots of cubic equations. It is quite complicated, however, which is why we do not use it. More information can be found here: <http://math.vanderbilt.edu/schectex/courses/cubic>

Additional comments regarding the Dawkins reading:

Dawkins has several examples where he asks for an “approximation to six decimal places.” This is another great way to guarantee the decimal place accuracy that we desire.

Example using TI-Nspire, TI-84, and Google Sheets:



	A	B	C	D
1	x_n	f(x_n)	f'(x_n)	x_(n+1)
2	1	=A2^2-2	=2*A2	=A2-B2/C2
3	=A2-B2/C2	=A3^2-2	=2*A3	=A3-B3/C3
4	=A3-B3/C3	=A4^2-2	=2*A4	=A4-B4/C4
5	=D4	=A5^2-2	=2*A5	=A5-B5/C5

	A	B	C	D
1	x_n	f(x_n)	f'(x_n)	x_(n+1)
2	1	-1	2	1.5
3	1.5	0.25	3	1.416666667
4	1.416666667	0.006944444444	2.833333333	1.414215686
5	1.414215686	0.000006007304	2.828431373	1.414213562