Directed Study in Mathematics — Name ______________________

1.3 Applied Optimization Problems — Date ______________ Pd ______

Required reading:

- Larson 9e: pages 218-222
- Dawkins: Calculus I, section 4-8: Optimization
  http://tutorial.math.lamar.edu/Classes/CalcI/Optimization.aspx
  - Notes: Read all. (Last modified: 05/30/2018)
  - Practice Problems: Review all. (Last modified: 02/19/2018)
- Dawkins: Calculus I, section 4-9: More Optimization
  http://tutorial.math.lamar.edu/Classes/CalcI/MoreOptimization.aspx
  - Notes: Read all. (Last modified: 05/30/2018)
  - Practice Problems: Review all. (Last modified: 06/04/2018)

Required homework:

- Larson 9e: page 224-227, problems 25, 39, 54
- Dawkins: Section 4-8, Assignment problems 20, 21 (Last modified: 01/22/2019)
- Dawkins: Section 4-9, Assigned problems 11, 15, 18 (Last modified: 06/04/2018)

Additional comments regarding this topic:

Applied optimization problems is nothing new, we saw this topic in AP Calculus BC. Some of the assigned homework problems, however, might be a bit different from what we previously saw. The purpose of this lesson is simply to give us more practice with solving a greater variety fo applied optimization problems.

Additional comments regarding the Larson reading:

When we studied applied optimization problems in AP Calculus BC, we usually used the First Derivative Test for Absolute Extrema. Take note, however, that Larson never defines the FDT for Absolute Extrema anywhere in his text—if we look back at our AP Calculus BC notes, we will see this version of the FDT actually comes from Dawkins' website. As such, Larson sometimes does things a bit differently from what we might expect.

Example 1

For instance, Larson utilizes a closed interval $[0, \sqrt{108}]$ for a domain restriction. But when $x = 0$, this yields a volume $V = 0$. Similarly, when $x = \sqrt{108}$, this yields a volume $V = 0$. Mathematically, this is not too big an issue. But in AP Calculus BC, we would have said that we want to consider a box with positive volume, so we should exclude these x-values from the domain. If we do that, we would not be able to use the Candidates Test to justify the absolute maximum; we would have to use the FDT for Absolute Extrema.

Examples 4 and 5

That said, in these examples, we *do* want closed intervals for those domain restrictions. As such, we can use the Candidates Test to justify the absolute extrema; we do not have to use the FDT for Absolute Extrema.

Examples 2 and 3

Larson gets a little "lazy" here—he doesn't justify the extrema adequately, choosing to use the First Derivative Test—but the regular FDT only acts to justify relative extrema. We need to give a more global argument, using the FDT for Absolute Extrema.

An issue for Example 2, however, is that there are three internal critical numbers, and the FDT for Absolute Extrema is written in a way that expects only one critical number. We can rectify this, however, by noting the symmetry of the graph of $y = 4 - x^2$ and the symmetry of the primary equation $d = \sqrt{x^4 - 3x^2 + 4}$. If we restrict the domain to $[0, \infty)$, we now have a situation where there is only one interior candidate to consider, which allows us to use the FDT for Absolute Extrema. We just have to note that the *opposite* value of x that we find in this problem also works as an absolute minimum when considering the unrestricted domain $(-\infty, \infty)$.

Additional comments regarding the Dawkins reading:

Section 4-8, Example 1

Like with Larson Example 1, Dawkins uses a closed interval $[0, 250]$ for his domain restriction. Unlike Larson, Dawkins actually mentions this issue, choosing not to make a big deal of it. But in the subsequent paragraphs, he shows more concern for endpoint treatment, and this discussion leads to his introduction of the First Derivative Test for Absolute Extrema.

After that, Dawkins introduces the Second Derivative Test for Absolute Extrema. We did not include this in AP Calculus BC because many textbooks do not utilize an SDT for Absolute Extrema, and it does involve additional work compared to the FDT for Absolute Extrema. But since we see it here now, it is fair game to use.

Section 4-8, Example 3

Dawkins fails to recognize that there is an upper restriction on the domain: w must be less than $\sqrt{5}$. (Consider the constraint equation $2w^2 + 4wh = 10$. If we make h very small in magnitude, we end up with an inequality $2w^2 < 10$, which becomes $0 < w < \sqrt{5}$.

Section 4-8, Example 5 and Section 4-9, Example 2

Note that Dawkins uses a closed interval for his domain restriction once again, when we would usually use an open interval.

Section 4-9, Example 3

Dawkins' *Solution 1* is similar to Larson's Example 2. He tries to justify the absolute extrema by considering several different cases, and this is a perfectly valid strategy. But the symmetry argument mentioned on the previous page of this handout might be easier to deal with.