Name Answer Key

Date ____________ Pd ______

AP Calculus BC: Gateway Exam #4 (20 min)
Passing Score = 75% Correct

1. **The Logistic Differential Equation**: $\frac{dy}{dt} = ky\left(1 - \frac{y}{L}\right)$

   The Logistic Equation: $y = \frac{L}{1 + be^{-kt}}$

2. **Area of a Region Between Two Curves**:

   If f and g are continuous on $[a, b]$ and $g(x) \le f(x)$ for all x in $[a, b]$, then the area of the region bounded by the graphs of f and g and the vertical lines $x = a$ and $x = b$ is: $A = \int_a^b \left(f(x) - g(x)\right) dx$

3. **The Washer Method**: To find the volume of a solid of revolution with the washer method, use one of the following:

   Horizontal axis of revolution on $[a, b]$: $V = \pi\int_a^b \left(\left(R(x)\right)^2 - \left(r(x)\right)^2\right) dx$

   Vertical axis of revolution on $[c, d]$: $V = \pi\int_c^d \left(\left(R(y)\right)^2 - \left(r(y)\right)^2\right) dy$

4. **Volumes of Solids with Known Cross Sections**:

   On $[a, b]$, for cross sections of area $A(x)$ taken perpendicular to the x-axis: $V = \int_a^b A(x)\, dx$

   On $[c, d]$, for cross sections of area $A(y)$ taken perpendicular to the y-axis: $V = \int_c^d A(y)\, dy$

5. **Definition of Arc Length**:

   For a smooth curve $y = f(x)$, the arc length of f between a and b is: $s = \int_a^b \sqrt{1 + \left(f'(x)\right)^2}\, dx$

   For a smooth curve $x = g(y)$, the arc length of g between c and d is: $s = \int_c^d \sqrt{1 + \left(g'(y)\right)^2}\, dy$

6. **Integration by Parts**:

   If u and v are functions of x and have continuous derivatives, then: $\int u\, dv = uv - \int v\, du$

7. **L'Hôpital's Rule**:

   Let f and g be functions that are differentiable on an interval (a, b) containing c, except possibly at c itself.

   Assume that $g'(x) \ne 0$ for all x in (a, b), except possibly at c itself.

   If $\lim_{x\to c} f(x) = 0$ and $\lim_{x\to c} g(x) = 0$ (OR) $\lim_{x\to c} f(x) = -\infty \text{ or } \infty$ and $\lim_{x\to c} g(x) = -\infty \text{ or } \infty$,

   then $\lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{f'(x)}{g'(x)}$ provided the limit on the right exists (or is infinite).

8. **The *n*th-Term Test**: If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=N}^{\infty} a_n$ diverges.

9. **Geometric Series**: A geometric series with ratio r diverges if $|r| \geq 1$.

   If $0 < |r| < 1$, then the series converges to the sum $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$.

10. **Definition of *n*th Taylor Polynomial**: If f has n derivatives at c, then the nth Taylor polynomial for f at c is:

$$P_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \cdots + \frac{f^{(n)}(c)}{n!}(x-c)^n$$

11. **Lagrange Error Bound**: If $\max\left|f^{(n+1)}(z)\right|$ is the maximum value of $f^{(n+1)}(z)$ between x and c, then:

$$|R_n(x)| \leq \frac{\max\left|f^{(n+1)}(z)\right|}{(n+1)!}|x-c|^{n+1}$$

12. **Definition of Power Series**:

   If x is a variable, then an infinite series of the form below is called a power series centered at c:

$$\sum_{n=0}^{\infty} a_n(x-c)^n = a_0 + a_1(x-c) + a_2(x-c)^2 + \cdots + a_n(x-c)^n + \cdots$$

13. **The Ratio Test**: Let $\sum_{n=N}^{\infty} a_n$ be a series with nonzero terms. $\sum_{n=N}^{\infty} a_n$ converges absolutely if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| < 1$.

   $\sum_{n=N}^{\infty} a_n$ diverges if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| > 1$ or $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \infty$.

   The Ratio Test is inconclusive if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1$.

14. **The Integral Test**: If f is positive, continuous, and decreasing for $x \geq N$ and $a_n = f(n)$,

   then $\sum_{n=N}^{\infty} a_n$ and $\int_N^{\infty} f(x)\,dx$ either both converge or both diverge.

15. ***p*-Series**: The p-series $\sum_{n=N}^{\infty} \frac{1}{n^p}$ converges if $p > 1$, and diverges if $0 < p \leq 1$.

16. **Direct Comparison Test**: Let $0 < a_n \leq b_n$ for all n.

   If $\sum_{n=N}^{\infty} b_n$ converges, then $\sum_{n=N}^{\infty} a_n$ converges. If $\sum_{n=N}^{\infty} a_n$ diverges, then $\sum_{n=N}^{\infty} b_n$ diverges.

17. **Limit Comparison Test**:

Suppose that $a_n > 0$, $b_n > 0$, and $\lim_{n\to\infty} \frac{a_n}{b_n} = L$, where L is finite and positive.

Then the two series $\sum_{n=N}^{\infty} a_n$ and $\sum_{n=N}^{\infty} b_n$ either both converge or both diverge.

18. **Alternating Series Test**: Let $a_n > 0$.

The alternating series $\sum_{n=N}^{\infty} (-1)^n a_n$ and $\sum_{n=N}^{\infty} (-1)^{n+1} a_n$ converge if the following two conditions are met:

(1) $\lim_{n\to\infty} a_n = 0$ and (2) $a_{n+1} \le a_n$ for all $n \ge N$

19. **Alternating Series Remainder**: Let $a_n > 0$. If an alternating series $\sum_{n=0}^{\infty} (-1)^n a_n$ converges to S,

and S_N is used to approximate the sum, then: $|S - S_N| \le a_{N+1}$

20. **Definitions of Absolute and Conditional Convergence**:

$\sum_{n=N}^{\infty} a_n$ is absolutely convergent if $\sum_{n=N}^{\infty} |a_n|$ converges.

$\sum_{n=N}^{\infty} a_n$ is conditionally convergent if $\sum_{n=N}^{\infty} a_n$ converges but $\sum_{n=N}^{\infty} |a_n|$ diverges.

21. **Absolute Convergence**: If the series $\sum_{n=N}^{\infty} |a_n|$ converges, then the series $\sum_{n=N}^{\infty} a_n$ also converges.

22. **Definition of Taylor Series**:

If a function f has derivatives of all orders at $x = c$, then the series below is called the Taylor series for $f(x)$ at c.

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \cdots + \frac{f^{(n)}(c)}{n!}(x-c)^n + \cdots$$

23. **Power Series for Elementary Functions**: $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \cdots$

24. **Power Series for Elementary Functions**: $\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \cdots$

25. **Power Series for Elementary Functions**: $\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + \frac{(-1)^n x^{2n}}{(2n)!} + \cdots$

26. **Power Series for Elementary Functions**: $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + \cdots + x^n + \cdots$

27. **Slope in Polar Form**: Let $r(\theta)$ be a differentiable function, with $x(\theta) = \underline{r\cos(\theta)}$ and $y(\theta) = \underline{r\sin(\theta)}$.

The slope of the tangent line to the graph of $r(\theta)$ at the point $(r,\ \theta)$ is $\underline{\dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta}}$ provided that

$\underline{\dfrac{dx}{d\theta} \neq 0}$ at $(r,\ \theta)$.

28. **Area in Polar Coordinates**:

If f is continuous and nonnegative on the interval $[\alpha,\ \beta]$, $0 < \beta - a < 2\pi$, then the area of the region bounded by the graph of $r = f(\theta)$ between the radial lines $\theta = \alpha$ and $\theta = \beta$ is given by: $A = \underline{\dfrac{1}{2}\int_{\alpha}^{\beta} r^2\, d\theta}$

29. **Parametric Form of the Derivative**: If a smooth curve C is given by the equations $x = f(t)$ and $y = g(t)$, then

$\dfrac{dy}{dx} = \underline{\dfrac{dy/dt}{dx/dt}}$ and $\dfrac{d^2y}{dx^2} = \underline{\dfrac{\frac{d}{dt}\left[\frac{dy}{dx}\right]}{dx/dt}}$, $\underline{\dfrac{dx}{dt} \neq 0}$.

30. **Arc Length in Parametric Form**: If a smooth curve C is given by $x = f(t)$ and $y = g(t)$ such that C does not intersect itself on the interval $a \le t \le b$ (except possibly at the endpoints), then the arc length of C over the interval is given by: $s = \underline{\int_a^b \sqrt{\left(f'(t)\right)^2 + \left(g'(t)\right)^2}\, dt}$

31. **Definitions of Velocity and Acceleration**: If x and y are twice-differentiable functions of t. and $\mathbf{r}$ is a position vector given by $\mathbf{r}(t) = \langle x(t),\ y(t)\rangle$, then the velocity vector, acceleration vector, and speed at time t are as follows:

velocity: $\underline{\vec{v}(t)} = \underline{\vec{r}'(t)} = \underline{\langle x'(t), y'(t)\rangle}$

acceleration: $\underline{\vec{a}(t)} = \underline{\vec{r}''(t)} = \underline{\langle x''(t), y''(t)\rangle}$

speed: $\underline{\|\vec{v}(t)\|} = \underline{\|\vec{r}'(t)\|} = \underline{\sqrt{\left(x'(t)\right)^2 + \left(y'(t)\right)^2}}$

32. **Distance and Total Displacement**:

Let $\underline{\vec{v}(t) = \langle x'(t), y'(t)\rangle}$ be the velocity of a particle, where $\underline{x'(t)}$ and $\underline{y'(t)}$ are continuous on $[a,\ b]$.

The displacement of the particle on $[a,\ b]$ is: $\underline{\int_a^b \vec{v}(t)\, dt = \left\langle \int_a^b x'(t)\, dt,\ \int_a^b y'(t)\, dt \right\rangle}$

The total distance traveled by the particle on $[a,\ b]$ is: $\underline{\int_a^b \|\vec{v}(t)\|\, dt = \int_a^b \sqrt{\left(x'(t)\right)^2 + \left(y'(t)\right)^2}\, dt}$

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1. **The Logistic Differential Equation**: ____________

   The Logistic Equation: ____________

2. **Area of a Region Between Two Curves**:

   If f and g are continuous on $[a, b]$ and $g(x) \le f(x)$ for all x in $[a, b]$, then the area of the region bounded by the graphs of f and g and the vertical lines $x = a$ and $x = b$ is: $A =$ ______________

3. **The Washer Method**: To find the volume of a solid of revolution with the washer method, use one of the following:

   Horizontal axis of revolution on $[a, b]$: $V =$ ____________________

   Vertical axis of revolution on $[c, d]$: $V =$ ____________________

4. **Volumes of Solids with Known Cross Sections**:

   On $[a, b]$, for cross sections of area _____ taken perpendicular to the x-axis: $V =$ _________

   On $[c, d]$, for cross sections of area _____ taken perpendicular to the y-axis: $V =$ _________

5. **Definition of Arc Length**:

   For a smooth curve $y = f(x)$, the arc length of f between a and b is: $s =$ ______________

   For a smooth curve $x = g(y)$, the arc length of g between c and d is: $s =$ ______________

6. **Integration by Parts**:

   If u and v are functions of x and have continuous derivatives, then: ______________

7. **L'Hôpital's Rule**:

   Let f and g be functions that are differentiable on an interval (a, b) containing c, except possibly at c itself. Assume that ________ for all x in (a, b), except possibly at c itself.

   If __________ and __________ (OR) ______________ and ______________,

   then ________________ provided the limit on the right exists (or is infinite).

8. **The *n*th-Term Test**: If ____________, then $\sum_{n=N}^{\infty} a_n$ __________.

9. **Geometric Series**: A geometric series with ratio r diverges if __________.

If __________, then the series converges to the sum $\sum_{n=0}^{\infty} ar^n =$ ________.

10. **Definition of *n*th Taylor Polynomial**: If f has n derivatives at c, then the nth Taylor polynomial for f at c is:

$P_n(x) =$ ____

11. **Lagrange Error Bound**: If ______________ is the maximum value of _________ between x and c, then:

$|R_n(x)| \leq$ _____________________

12. **Definition of Power Series**:

If x is a variable, then an infinite series of the form below is called a power series centered at c:

____

13. **The Ratio Test**: Let $\sum_{n=N}^{\infty} a_n$ be a series with nonzero terms. $\sum_{n=N}^{\infty} a_n$ ___________________ if _____________.

$\sum_{n=N}^{\infty} a_n$ ___________________ if _____________ or _____________.

The Ratio Test is inconclusive if _____________.

14. **The Integral Test**: If f is ___________, ___________, and ___________ for $x \geq N$ and $a_n = f(n)$,

then ___________ and ___________ either ______________ or ______________.

15. ***p*-Series**: The p-series ________ converges if _________, and diverges if _________.

16. **Direct Comparison Test**: Let $0 < a_n \leq b_n$ for all n.

If $\sum_{n=N}^{\infty} b_n$ __________, then $\sum_{n=N}^{\infty} a_n$ __________. If $\sum_{n=N}^{\infty} a_n$ __________, then $\sum_{n=N}^{\infty} b_n$ __________.

17. **Limit Comparison Test**:

Suppose that ________ , ________ , and ____________ , where ____ is __________ and __________ .

Then the two series $\sum_{n=N}^{\infty} a_n$ and $\sum_{n=N}^{\infty} b_n$ either ______________ or ______________ .

18. **Alternating Series Test**: Let $a_n > 0$.

The alternating series $\sum_{n=N}^{\infty} (-1)^n a_n$ and $\sum_{n=N}^{\infty} (-1)^{n+1} a_n$ __________ if the following two conditions are met:

(1) ____________________ and (2) ____________________

19. **Alternating Series Remainder**: Let $a_n > 0$. If an alternating series $\sum_{n=0}^{\infty} (-1)^n a_n$ converges to S,

and S_N is used to approximate the sum, then: ______________

20. **Definitions of Absolute and Conditional Convergence**:

$\sum_{n=N}^{\infty} a_n$ is ______________________ if ________ converges.

$\sum_{n=N}^{\infty} a_n$ is ______________________ if ________ converges but ________ diverges.

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22. **Definition of Taylor Series**:

If a function f has derivatives of all orders at $x = c$, then the series below is called the Taylor series for $f(x)$ at c.

____

23. **Power Series for Elementary Functions**: $e^x =$ ____

24. **Power Series for Elementary Functions**: $\sin(x) =$ ____

25. **Power Series for Elementary Functions**: $\cos(x) =$ ____

26. **Power Series for Elementary Functions**: $\frac{1}{1-x} =$ ____

27. **Slope in Polar Form**: Let $r(\theta)$ be a differentiable function, with $x(\theta) =$ ________ and $y(\theta) =$ ________.

The slope of the tangent line to the graph of $r(\theta)$ at the point $(r,\ \theta)$ is ____________ provided that

________ at $(r,\ \theta)$.

28. **Area in Polar Coordinates**:

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$\dfrac{dy}{dx} =$ __________ and $\dfrac{d^2y}{dx^2} =$ __________, ________.

30. **Arc Length in Parametric Form**: If a smooth curve C is given by $x = f(t)$ and $y = g(t)$ such that C does not intersect itself on the interval $a \le t \le b$ (except possibly at the endpoints), then the arc length of C over the interval is given by: $s =$ ______________________

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velocity: ________ = ________ = __________________

acceleration: ________ = ________ = __________________

speed: ________ = ________ = __________________

32. **Distance and Total Displacement**:

Let ________________ be the velocity of a particle, where ______ and ______ are continuous on $[a,\ b]$.

The displacement of the particle on $[a,\ b]$ is: ________________________________

The total distance traveled by the particle on $[a,\ b]$ is: ________________________________