Name Answer Key

Date ____________ Pd ______

AP Calculus BC: Gateway Exam #3 (20 min)
Passing Score = 75% Correct

1. **Definition of Continuity at a Point**: A function f is continuous at c if $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = f(c)$.

2. **The Intermediate Value Theorem**:

   If f is continuous on the interval $[a, b]$, and k is any number between $f(a)$ and $f(b)$,

   then there is at least one number c in $[a, b]$ such that $f(c) = k$.

3. **Definition of Horizontal Asymptote**:

   The line $y = L$ is a horizontal asymptote of the graph of f if $\lim_{x\to -\infty} f(x) = L$ or $\lim_{x\to \infty} f(x) = L$.

4. **Definitions of the Derivative of a Function**:

   $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ (original) and $f'(c) = \lim_{x\to c} \frac{f(x) - f(c)}{x - c}$ (alternative)

5. **Continuity and Differentiability**: If f is differentiable at $x = c$, then f is continuous at $x = c$.

6. **Average Rate of Change**: The average rate of change of a function f on an interval $[a, b]$ is $\frac{f(b) - f(a)}{b - a}$.

For Problems 7-9, $s(t)$ represents the position of a particle at time t, $v(t)$ represents the instantaneous velocity of the particle at time t, and $a(t)$ represents the instantaneous acceleration of the particle at time t.

7. **Particle Motion (Average Rates of Change)**:

   Average velocity on $[a, b]$: $\frac{s(b) - s(a)}{b - a}$ and average acceleration on $[a, b]$: $\frac{v(b) - v(a)}{b - a} = \frac{s'(b) - s'(a)}{b - a}$

8. **Particle Motion (Instantaneous Rates of Change)**:

   Velocity $v(t) = s'(t)$, speed $|v(t)| = |s'(t)|$, and acceleration $a(t) = v'(t) = s''(t)$

9. **Particle Motion (Speeding Up and Slowing Down)**:

   The particle is speeding up at time $t = c$ if $v(c)$ and $a(c)$ have the same sign .

   The particle is slowing down at time $t = c$ if $v(c)$ and $a(c)$ have opposite signs .

10. **The Extreme Value Theorem**: If f is continuous on an interval $[a, b]$,

    then f has both a minimum and a maximum on the interval.

11. **Definition of a Critical Number**: Let f be defined at c.

If $f'(c)=0$ or if f is not differentiable at c, then c is a critical number of f.

12. **The Mean Value Theorem**:

If f is continuous on the interval $[a, b]$ and differentiable on the interval (a, b),

then there exists a number c in (a, b) such that $f'(c)=\dfrac{f(b)-f(a)}{b-a}$.

13. **Test for Increasing and Decreasing Functions**: Let f be continuous on the interval $[a, b]$.

If $f'(x)>0$ for all x in (a, b), then f is increasing on $[a, b]$.

If $f'(x)<0$ for all x in (a, b), then f is decreasing on $[a, b]$.

14. **The First Derivative Test**:

If $f'(x)$ changes from negative to positive at c, then f has a relative minimum at $x=c$.

If $f'(x)$ changes from positive to negative at c, then f has a relative maximum at $x=c$.

15. **Definition of Concavity**:

The graph of f is concave upward on (a, b) if $f'(x)$ is increasing on (a, b).

The graph of f is concave downward on (a, b) if $f'(x)$ is decreasing on (a, b).

16. **Test for Concavity with the Second Derivative**:

The graph of f is concave upward on (a, b) if $f''(x)>0$ for all x in (a, b).

The graph of f is concave downward on (a, b) if $f''(x)<0$ for all x in (a, b).

17. **Definition of Point of Inflection**:

There is a point of inflection of the graph of f at $x=c$ if the concavity of f changes

from upward to downward (or downward to upward) at the point.

18. **Test for Point of Inflection with the First Derivative**:

There is a point of inflection of the graph of f at $x=c$ if $f'(x)$ changes

from increasing to decreasing (or decreasing to increasing) at the point.

19. **The Second Derivative Test**: Let f be a function such that $f'(c)=0$.

If $f''(c)>0$, then f has a relative minimum at $x=c$.

If $f''(c)<0$, then f has a relative maximum at $x=c$.

20. **The First Derivative Test for Absolute Extrema**:

If $f'(x)<0$ for all $x<c$ and $f'(x)>0$ for all $x>c$, then f has an absolute minimum at $x=c$.

If $f'(x)>0$ for all $x<c$ and $f'(x)<0$ for all $x>c$, then f has an absolute maximum at $x=c$.

21. **Definition of Antiderivative**:

A function F is an antiderivative of f on an interval I if $\underline{F'(x) = f(x)}$ for all x in I.

22. **Left Riemann Sums**:

A left Riemann sum approximation of $\int_a^b f(x)\,dx$ is __less than__ the actual value when f is __increasing__, and __greater than__ the actual value when f is __decreasing__.

23. **Right Riemann Sums**:

A right Riemann sum approximation of $\int_a^b f(x)\,dx$ is __greater than__ the actual value when f is __increasing__, and __less than__ the actual value when f is __decreasing__.

24. **Trapezoidal Approximations**:

A trapezoidal approximation of $\int_a^b f(x)\,dx$ is __greater than__ the actual value when the graph of f is __concave upward__, and __less than__ the actual value when the graph of f is __concave downward__.

25. **The First Fundamental Theorem of Calculus**: If a function f is __continuous__ on the interval $\underline{[a, b]}$ and F is an __antiderivative__ of f on the interval $\underline{[a, b]}$, then $\underline{\int_a^b f(x)\,dx = F(b) - F(a)}$.

26. **Definition of the Average Value of a Function on an Interval**:

If f is integrable on the interval $\underline{[a, b]}$, then the average value of f on the interval is $\underline{\frac{1}{b-a}\int_a^b f(x)\,dx}$.

27. **The Second Fundamental Theorem of Calculus**:

If f is __continuous__ on an interval I containing a, then, for every x in the interval, $\underline{\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)}$.

28. **The Net Change Theorem**: If $\underline{F'(x)}$ is a rate of change of a quantity $\underline{F(x)}$, then the total (or net) change of $\underline{F(x)}$ on the interval $\underline{[a, b]}$ is given by $\underline{\int_a^b F'(x)\,dx = F(b) - F(a)}$.

For Problems 29-30, $s(t)$ represents the position of a particle at time t, $v(t)$ represents the instantaneous velocity of the particle at time t, and $a(t)$ represents the instantaneous acceleration of the particle at time t.

29. **Particle Motion (Integration)**:

Average velocity on $[c, d]$: $\underline{\frac{1}{d-c}\int_c^d v(t)\,dt}$ and average acceleration on $[c, d]$: $\underline{\frac{1}{d-c}\int_c^d a(t)\,dt}$

30. **Particle Motion (Displacement vs. Total Distance)**:

Displacement on $[c, d]$: $\underline{\int_c^d v(t)\,dt}$ and total distance traveled on $[c, d]$: $\underline{\int_c^d |v(t)|\,dt}$

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1. **Definition of Continuity at a Point**: A function f is continuous at c if ______________________.

2. **The Intermediate Value Theorem**:

   If f is __________ on the interval ______, and k is any number between ______ and ______, then there is at least one number c in ______ such that ________.

3. **Definition of Horizontal Asymptote**:

   The line ______ is a horizontal asymptote of the graph of f if ____________ or ____________.

4. **Definitions of the Derivative of a Function**:

   $f'(x) =$ ________________ (original) and $f'(c) =$ ________________ (alternative)

5. **Continuity and Differentiability**: If f is ____________ at $x = c$, then f is ____________ at $x = c$.

6. **Average Rate of Change**: The average rate of change of a function f on an interval $[a, b]$ is ____________.

For Problems 7-9, $s(t)$ represents the position of a particle at time t, $v(t)$ represents the instantaneous velocity of the particle at time t, and $a(t)$ represents the instantaneous acceleration of the particle at time t.

7. **Particle Motion (Average Rates of Change)**:

   Average velocity on $[a, b]$: __________ and average acceleration on $[a, b]$: ____________________

8. **Particle Motion (Instantaneous Rates of Change)**:

   Velocity $v(t) =$ ______, speed ____________, and acceleration $a(t) =$ __________

9. **Particle Motion (Speeding Up and Slowing Down)**:

   The particle is speeding up at time $t = c$ if ______ and ______ have ____________.

   The particle is slowing down at time $t = c$ if ______ and ______ have ____________.

10. **The Extreme Value Theorem**: If f is __________ on an interval ______,

    then f has both a __________ and a __________ on the interval.

11. **Definition of a Critical Number**: Let f be defined at c.

If __________ or if f is __________________ at c, then c is a critical number of f.

12. **The Mean Value Theorem**:

If f is ______________ on the interval ________ and ______________ on the interval ________,

then there exists a number c in ________ such that ____________________.

13. **Test for Increasing and Decreasing Functions**: Let f be ____________ on the interval ________.

If __________ for all x in (a, b), then f is increasing on ________.

If __________ for all x in (a, b), then f is decreasing on ________.

14. **The First Derivative Test**:

If _______ changes from __________ to __________ at c, then f has a relative minimum at $x = c$.

If _______ changes from __________ to __________ at c, then f has a relative maximum at $x = c$.

15. **Definition of Concavity**:

The graph of f is concave upward on (a, b) if _______ is ____________ on (a, b).

The graph of f is concave downward on (a, b) if _______ is ____________ on (a, b).

16. **Test for Concavity with the Second Derivative**:

The graph of f is concave upward on (a, b) if __________ for all x in (a, b).

The graph of f is concave downward on (a, b) if __________ for all x in (a, b).

17. **Definition of Point of Inflection**:

There is a point of inflection of the graph of f at $x = c$ if the ___________ of f changes

from ____________ to ____________ (or ____________ to ____________) at the point.

18. **Test for Point of Inflection with the First Derivative**:

There is a point of inflection of the graph of f at $x = c$ if _______ changes

from ____________ to ____________ (or ____________ to ____________) at the point.

19. **The Second Derivative Test**: Let f be a function such that __________.

If __________, then f has a relative minimum at $x = c$.

If __________, then f has a relative maximum at $x = c$.

20. **The First Derivative Test for Absolute Extrema**:

If __________ for all $x < c$ and __________ for all $x > c$, then f has an absolute minimum at $x = c$.

If __________ for all $x < c$ and __________ for all $x > c$, then f has an absolute maximum at $x = c$.

21. **Definition of Antiderivative**:

A function F is an antiderivative of f on an interval I if ____________ for all x in I.

22. **Left Riemann Sums**:

A left Riemann sum approximation of $\int_a^b f(x)\,dx$ is ____________ the actual value when f is ____________, and ____________ the actual value when f is ____________.

23. **Right Riemann Sums**:

A right Riemann sum approximation of $\int_a^b f(x)\,dx$ is ____________ the actual value when f is ____________, and ____________ the actual value when f is ____________.

24. **Trapezoidal Approximations**:

A trapezoidal approximation of $\int_a^b f(x)\,dx$ is ____________ the actual value when the graph of f is ________________, and ____________ the actual value when the graph of f is ________________.

25. **The First Fundamental Theorem of Calculus**: If a function f is ____________ on the interval ________ and F is an ____________ of f on the interval ________, then ____________________.

26. **Definition of the Average Value of a Function on an Interval**:

If f is integrable on the interval ________, then the average value of f on the interval is ________________.

27. **The Second Fundamental Theorem of Calculus**:

If f is ____________ on an interval I containing a, then, for every x in the interval, ____________________.

28. **The Net Change Theorem**: If ________ is a rate of change of a quantity ________, then the total (or net) change of ________ on the interval ________ is given by ____________________.

For Problems 29-30, $s(t)$ represents the position of a particle at time t, $v(t)$ represents the instantaneous velocity of the particle at time t, and $a(t)$ represents the instantaneous acceleration of the particle at time t.

29. **Particle Motion (Integration)**:

Average velocity on $[c, d]$: ________________ and average acceleration on $[c, d]$: ________________

30. **Particle Motion (Displacement vs. Total Distance)**:

Displacement on $[c, d]$: ____________ and total distance traveled on $[c, d]$: ____________