§9.4b Polar Coordinates and Differentiation

Slope and Tangent Lines

Notes based on: *Calculus for AP* by Larson & Battaglia. © 2017 Cengage Learning.
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Learning Goals: Students will be able to…

- Find the slope of a tangent line to a polar graph.

Slope and Tangent Lines

To find the slope of a tangent line to a polar graph, consider a differentiable function given by $r = f(\theta)$.

To find the slope in polar form, use the equations $x = r\cos(\theta) = f(\theta)\cos(\theta)$ and $y = r\sin(\theta) = f(\theta)\sin(\theta)$.

Find $x'(\theta)$ and $y'(\theta)$, then use the equation $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{y'(\theta)}{x'(\theta)}$ to determine $\frac{dy}{dx}$.

Slope and Tangent Lines

THEOREM SLOPE IN POLAR FORM

If f is a differentiable function of θ, then the *slope* of the tangent line to the graph of $r = f(\theta)$ at the point (r, θ) is

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{f(\theta)\cos\theta + f'(\theta)\sin\theta}{-f(\theta)\sin\theta + f'(\theta)\cos\theta}$$

provided that $dx/d\theta \neq 0$ at (r, θ).

This is based on the Product Rule with $x = f(\theta)\cos(\theta)$ and $y = f(\theta)\sin(\theta)$.

Slope and Tangent Lines

From this theorem, we can make the following observations:

1. Solutions of $\frac{dy}{d\theta} = 0$ yield horizontal tangents, provided that $\frac{dx}{d\theta} \neq 0$.
2. Solutions of $\frac{dx}{d\theta} = 0$ yield vertical tangents, provided that $\frac{dy}{d\theta} \neq 0$.

If $dy/d\theta$ and $dx/d\theta$ are *simultaneously* 0, then no conclusion can be drawn about tangent lines.

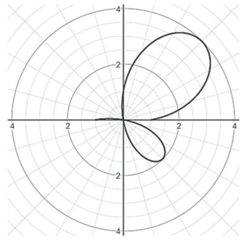
Example: Slope and Tangent Lines

Given $r = 2 - 2\sin(\theta)$, find dy/dx and the equation of the tangent line at $\theta = 0$.

Example: Slope and Tangent Lines

Let r be the function given by $r(\theta) = 1 + 3\sin(2\theta)$ for $0 \le \theta \le \pi$. The graph of r is shown in the figure.

For $0 \le \theta \le \pi$, find the value of θ that gives the point on the graph that is farthest from the origin. Justify your answer.



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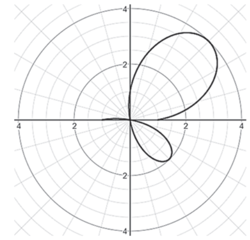

Example: Slope and Tangent Lines

The graphs of the polar curves $r = 3$ and $r = 1 + 2\sin(\theta)$ are shown in the figure.

(a) Write an expression for the distance D between the curves for $0 < \theta < \pi/2$.

(b) Find $D'(\pi/4)$. Explain the meaning of this value in the context of this problem.