

## §9.4b Polar Coordinates and Differentiation

## Slope and Tangent Lines

Notes based on: *Calculus for AP* by Larson & Battaglia. © 2017 Cengage Learning.  
*Calculus, AP Edition, 9th ed.* by Larson & Edwards. © 2010 Brooks/Cole, Cengage Learning.

**Learning Goals: Students will be able to...**

- Find the slope of a tangent line to a polar graph.

**Learning Objectives: Students will be able to...**

- 2.1C Calculate derivatives.  
 2.2A Use derivatives to analyze properties of a function.  
 2.3B Solve problems involving the slope of a tangent line.

## Slope and Tangent Lines

To find the slope of a tangent line to a polar graph, consider a differentiable function given by  $r = f(\theta)$ .

To find the slope in polar form, use the equations  $x = r \cos(\theta) = f(\theta) \cos(\theta)$  and  $y = r \sin(\theta) = f(\theta) \sin(\theta)$ .

Find  $x'(\theta)$  and  $y'(\theta)$ , then use the equation  $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{y'(\theta)}{x'(\theta)}$  to determine  $\frac{dy}{dx}$ .

## Slope and Tangent Lines

**THEOREM** SLOPE IN POLAR FORM

If  $f$  is a differentiable function of  $\theta$ , then the *slope* of the tangent line to the graph of  $r = f(\theta)$  at the point  $(r, \theta)$  is

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{f(\theta) \cos \theta + f'(\theta) \sin \theta}{-f(\theta) \sin \theta + f'(\theta) \cos \theta}$$

provided that  $dx/d\theta \neq 0$  at  $(r, \theta)$ .

This is based on the Product Rule with  $x = f(\theta) \cos(\theta)$  and  $y = f(\theta) \sin(\theta)$ .

## Slope and Tangent Lines

From this theorem, we can make the following observations:

1. Solutions of  $\frac{dy}{d\theta} = 0$  yield horizontal tangents, provided that  $\frac{dx}{d\theta} \neq 0$ .
2. Solutions of  $\frac{dx}{d\theta} = 0$  yield vertical tangents, provided that  $\frac{dy}{d\theta} \neq 0$ .

If  $dy/d\theta$  and  $dx/d\theta$  are *simultaneously* 0, then no conclusion can be drawn about tangent lines.

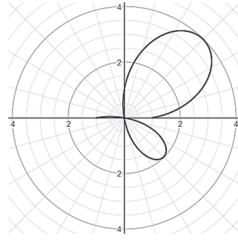
## Example: Slope and Tangent Lines

Given  $r = 2 - 2\sin(\theta)$ , find  $dy/dx$  and the equation of the tangent line at  $\theta = 0$ .

## Example: Slope and Tangent Lines

Let  $r$  be the function given by  $r(\theta) = 1 + 3\sin(2\theta)$  for  $0 \leq \theta \leq \pi$ . The graph of  $r$  is shown in the figure.

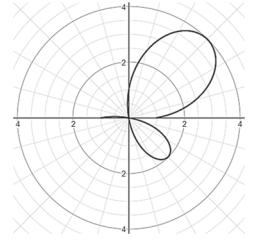
For  $0 \leq \theta \leq \pi$ , find the value of  $\theta$  that gives the point on the graph that is farthest from the origin. Justify your answer.



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## Example: Slope and Tangent Lines

The graphs of the polar curves  $r = 3$  and  $r = 1 + 2\sin(\theta)$  are shown in the figure.

- Write an expression for the distance  $D$  between the curves for  $0 < \theta < \pi/2$ .
- Find  $D'(\pi/4)$ . Explain the meaning of this value in the context of this problem.

