§8.10b Manipulation of Known Taylor Series

Deriving Taylor Series from a Basic List

Notes based on: *Calculus for AP* by Larson & Battaglia. © 2017 Cengage Learning.
Calculus, AP Edition, 9th ed. by Larson & Edwards. © 2010 Brooks/Cole, Cengage Learning.
Taylor Polynomials and Infinite Series by B. Goldstein. © 2015 Benjamin Goldstein.

Learning Goals: Students will be able to…

- Find a Taylor or Maclaurin series for a function.
- Use a basic list of Taylor series to find other Taylor series.

Learning Objectives: Students will be able to…

1.1A Express limits symbolically using correct notation, and interpret limits expressed symbolically.
1.1C Determine limits of functions.
2.1C Calculate derivatives.
2.1D Determine higher order derivatives.
3.1A Recognize antiderivatives of basic functions.
3.3B Calculate antiderivatives, and evaluate definite integrals.
4.1A *(BC: Determine whether a series converges or diverges.)*
4.1B *(BC: Determine or estimate the sum of a series.)*
4.2B *(BC: Write a power series representing a given function.)*

Deriving Taylor Series from a Basic List

The list below provides the power series for several elementary functions with the corresponding intervals of convergence.

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots + \frac{x^n}{n!} + \cdots \qquad -\infty < x < \infty$$

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \cdots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \cdots \qquad -\infty < x < \infty$$

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots + \frac{(-1)^n x^{2n}}{(2n)!} + \cdots \qquad -\infty < x < \infty$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + \cdots + x^n + \cdots \qquad -1 < x < 1$$

Deriving Taylor Series from a Basic List

The direct determination of Taylor or Maclaurin coefficients using successive differentiation can be difficult.

There exists a shortcut for finding the coefficients indirectly—using the coefficients of a known Taylor or Maclaurin series.

Example: Deriving Taylor Series from a Basic List

Find the Maclaurin series for the function $f(x) = \cos(x^2)$. Include the first four nonzero terms and the general term.

Example: Deriving Taylor Series from a Basic List

Find the Maclaurin series for the function $g(x) = \frac{d}{dx}\left[\cos(x^2)\right]$. Include the first four nonzero terms and the general term.

Example: Deriving Taylor Series from a Basic List

Find the Maclaurin series for the function $f(x)=\dfrac{x}{1-x^2}$. Include the first four nonzero terms and the general term.

Example: Deriving Taylor Series from a Basic List

Find the Maclaurin series for the function $h(x)=\int_0^x e^{-t^2}\,dt$. Include the first four nonzero terms and the general term. Then approximate the value of $h(1)$ with an error of less than $1/40$.

Example: Deriving Taylor Series from a Basic List

Find the Maclaurin series for the function $h(x)=\int_0^x e^{-t^2}\,dt$. Include the first four nonzero terms and the general term. Then approximate the value of $h(1)$ with an error of less than $1/40$.

Example: Deriving Taylor Series from a Basic List

Find the Maclaurin series for the function $h(x)=\int_0^x e^{-t^2}\,dt$. Include the first four nonzero terms and the general term. Then approximate the value of $h(1)$ with an error of less than $1/40$.